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On binomial operator representations of certain polynomial

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Abstract

Based on the technique used by M. A. Khan and A. K. Shukla [4] here finite series representations of bionomial partial differential operators have been used to establish operator representations of various polynomials not considered in the earlier mentioned paper. The results obtained are believed to be new.

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1. Introduction :

Recently in 2009, M. A. Khan and A. K. Shukla [4] evolved a new technique to give operator representations of certain polynomials. They give binomial and trinomial operator representation of certain polynomials. The aim of the presnt paper is to strengthen the technique evolved by obtaining binomial operator represenations of some more polynomials not considered in the above mentioned paper.

2. The definition, notations and results used :

In deriving the operational representations of various polynomials use has been made of the fact that

$$D^{\mu}x^{\lambda} = \frac{\Gamma(1+\lambda)}{\Gamma(1+\lambda-\mu)}x^{\lambda-\mu}, \qquad D \equiv \frac{d}{dx} \tag{2.1}$$

Where λ and μ ,$\lambda \geq \mu$ are arbitrory real numbers.
In particular, use has been made of the following results:

$$D^r\ e^{-x} \ = \ (-1)^r\ e^{-x} \tag{2.2}$$

$$D^r\ x^{-\alpha} \ = \ (\alpha)_r\ (-1)^r\ x^{-\alpha-r}, \alpha \textit{ is not an integer} \tag{2.3}$$

$$D^r\ x^{-\alpha-n} \ = \ (\alpha+n)_r\ (-1)^r\ x^{-\alpha-n-r} \tag{2.4}$$

$$D^{n-r}\ x^{\alpha-1+n} \ = \ \frac{(\alpha)_n}{(\alpha)_r}\ x^{\alpha-1+r} \tag{2.5}$$

$$D^{n-r}\ x^{-\alpha} \ = \ \frac{(\alpha)_n(-1)^n}{(1-\alpha-n)_r}\ x^{-\alpha-n+r}, \alpha \textit{ is not an integer} \tag{2.6}$$

Where n and r are denote positive integrs and

$$(\alpha)_n = \alpha\ (\alpha+1)\cdot\cdots\cdot(\alpha+n-1); \qquad (\alpha)_0 = 1$$

We also need the defenitions of the following polynomials interms of hypergeometric function and also their notations (see [1],[3], [9], [10]).

CESARO POLYNOMIALS

It is denoted by the symbol $g_n^{(s)}(x)$ and is defined as

$$(2.7) \qquad g_n^{(s)}(x) = \begin{pmatrix} s+n \\ n \end{pmatrix} {}_2F_1\left[\begin{matrix} -n, & 1\ ; \\ -s-n\ ; & \end{matrix}\ x\right]$$

SYLVESTER POLYNOMIALS

It is denoted by the symbol $\varphi_n(x)$ and is defined as

$$(2.8) \qquad \varphi_n(x) \ = \ \frac{x^n}{n!}\ {}_2F_0\left[\begin{matrix} -n, & x; \\ & -; \end{matrix}\ x^{-1}\right]$$

SHIVELY'S PSEUDO LAGUERRE POLYNOMIALS

It is denoted by the symbol $R_n(a,x)$ and is defined as

$$(2.9) \qquad R_n(a,x) = \frac{(a)_{2n}}{n!(a)_n}\ {}_1F_1\left[\begin{matrix} -n\ ; \\ a+n\ ; \end{matrix}\ x\right]$$

HERMITE POLYNOMIAL

It is denoted by the symbol $H_n(x)$ and is defined as

$$(2.10) \ H_n(x) = (2x)^n\ {}_2F_0\left[\begin{matrix} -\frac{1}{2}n, -\frac{1}{2}n+\frac{1}{2}\ ; \\ -; \end{matrix}\ \frac{1}{x^2}\right]$$

BEDIENT POLYNOMIALS

Bedient [2], in his study of some polynomials assosiated with Appell's F_2 and F_3 introduced

$$(2.11)\quad R_n(\beta,\gamma,x)=\frac{(\beta)_n(2x)^n}{n!}\,{}_3F_2\left[\begin{matrix}-\frac{n}{2},-\frac{n}{2}+\frac{1}{2}\,,\gamma-\beta;\\ \gamma,1-\beta-n\,;\end{matrix}\quad\frac{1}{x^2}\right]$$

(2.12)

$$G_n(\alpha,\beta;x)=\frac{(\alpha)_n(\beta)_n(2x)^n}{n!(\alpha+\beta)_n}\,{}_3F_2\left[\begin{matrix}-\frac{1}{2}n,-\frac{1}{2}n+\frac{1}{2},\ 1-\alpha-\beta-n;\\ 1-\alpha-n,1-\beta-n\,;\end{matrix}\quad\frac{1}{x^2}\right]$$

LOMMEL POLYNOMIAL

It is denoted by the symbol $R_{n,\nu}(\frac{1}{z})$ and is defined as

$$(2.13)\quad R_{n,\nu}\left(\frac{1}{z}\right)=(\nu)_n(2z)^n\,{}_2F_3\left[\begin{matrix}-\frac{1}{2}n,-\frac{1}{2}n+\frac{1}{2}\,;\\ \nu,-n\,,1-\nu-n;\end{matrix}\quad-\frac{1}{z^2}\right]$$

BRAFMAN POLYNOMIAL

It is denoted by the symbol $B_n^m[\alpha_1,\alpha_2\cdots\alpha_r,\ \ \beta_1,\beta_2\cdots\beta_s;x]$ and is defined as

(2.14)

$$B_n^m[\alpha_1,\alpha_2\cdot\cdot\alpha_r,\ \ \beta_1,\beta_2\cdot\cdot\beta_s;x]=\ {}_{m+r}F_s\left[\begin{matrix}\Delta(m;-n),\alpha_1,\alpha_2,\cdots\alpha_r;\\ \beta_1,\beta_2\cdots\beta_s\,;\end{matrix}\quad x\right]$$

Where $\Delta(m;n)$ stands for set of m parameters $\frac{n}{m},\frac{n+1}{m},$.....$\frac{n+m-1}{m}$

GOULD AND HOPPER POLYNOMIAL

It is denoted by the symbol $g_n^m(x,h)$ and is defined as

$$(2.15)\quad g_n^m(x,h)=x^n\ {}_mF_0\left[\begin{matrix}\Delta(m;-n);\\ -;\end{matrix}\quad h\left(\frac{-m}{x}\right)^m\right]$$

LAHIRI POLYNOMIAL

It is denoted by the symbol $H_{n,m,\nu}(x)$ and is defined as

$$(2.16)\quad H_{n,m,\nu}(x)=(\nu\ x)^n\ {}_mF_0\left[\begin{matrix}\Delta(m;-n);\\ -;\end{matrix}\quad -\left(\frac{-m}{\nu\ x}\right)^m\right]$$

MADHEKAR-THAKARE'S GENARALIZED HYPERGEOMETRIC POLYNOMIAL

It is denoted by the symbol $J_n^{(\alpha,\beta)}(x;k)$ and is defined as

(2.17)

$$J_n^{(\alpha,\beta)}(x;k) = \frac{(\alpha+1)_{kn}}{n!}\ {}_{k+1}F_n\left[\begin{matrix} -n,\ \Delta(k;\alpha+\beta+n+1); \\ \Delta(k;\alpha+1); \end{matrix}\ \left(\frac{1-x}{2}\right)^k\right]$$

SRIVASTAVA'S GENERALIZED HYPERGEOMETRIC POLYNOMIAL

It is denoted by the symbol $H_n^{(\alpha,\beta)}(x;m)$ and is defined as

$$H_n^{(\alpha,\beta)}(x;m) = \begin{pmatrix} \alpha+(\beta+1)n \\ n \end{pmatrix}\ {}_{p+m}F_{q+m}\left[\begin{matrix} \Delta(m;-n), & (a_p)\ ; \\ \Delta(m;\alpha+\beta\ n+1), & (b_q); \end{matrix}\ x\right]$$

(2.18)

3. Operational representations :

If $D_x \equiv \frac{\partial}{\partial x}$ *and* $D_y \equiv \frac{\partial}{\partial\ y}$, M. A. Khan and A. K. Shukla [4] wrote the binomial expansion for $(D_x + D_y)^n$ as

(3.1) $$(D_x+D_y)^n \equiv \sum_{r=0}^{n} {}^nC_r D_x^{n-r} D_y^r$$

where ${}^nC_r = \frac{n!}{r!(n-r)!}$. By writing the finite series on the right of (3.1) M.A. Khan and A.K. Shukla [4] wrote (3.1) also as

(3.2) $$(D_x+D_y)^n \equiv \sum_{r=0}^{n} {}^nC_r D_x^r D_y^{n-r}$$

If F(x,y) is afunction of x and y, they obtained the following from (3.1) and (3.2)

(3.3)

$$(D_x+D_y)^n F(x,y) \equiv \sum_{r=0}^{n} \frac{(-n)_r(-1)^r}{r!} D_x^{n-r} D_y^r\ F(x,y)$$

(3.4)
$$(D_x+D_y)^n F(x,y) \equiv \sum_{r=0}^{n} \frac{(-n)_r(-1)^r}{r!} D_x^r D_y^{n-r}\, F(x,y)$$

In particular , if $F(x,y) = f(x)g(y)$. M.A.Khan and A.K.Shukla [4] wrote (3.3) and (3.4) in the form

(3.5)
$$(D_x+D_y)^n f(x)g(y) \equiv \sum_{r=0}^{n} \frac{(-n)_r(-1)^r}{r!} D_x^{n-r}\, f(x) D_y^r\, g(y)$$

(3.6)
$$(D_x+D_y)^n f(x)g(y) \equiv \sum_{r=0}^{n} \frac{(-n)_r(-1)^r}{r!} D_x^r f(x) D_y^{n-r} g(y)$$

Now by taking special values of $f(x)$ and $g(y)$ in (3.5), we obtain the following partial differential operator representations of the polynomials given above :

(3.7)
$$(D_x+D_y)^n \left\{x^{-s-1}y^{-1}\right\} \;=\; (-1)^n\, n!\, x^{-s-1-n} y^{-1} g_n^{(s)}\left(\frac{x}{y}\right)$$

(3.8)
$$(1+D_yD_z)^n \left\{y^{-x} e^{\frac{-1}{x}z}\right\} = n!\, y^{-x} e^{\frac{-1}{x}z} x^{-n} \varphi_n\left(\frac{y}{x}\right)$$

(3.9)
$$(1-D_x)^n \left\{x^{a-1+2n}\right\} = n!\, x^{a-1+2n} R_n(a,x)$$

(3.10)
$$(D_u - D_xD_y)^n \left\{u^{-1} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}}\right\} \;= (-1)^n\, n!\, 2^{-n}\, u^{-n-1}\, x^{-\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}}\, H_n\left(\sqrt{\frac{xy}{u}}\,\right)$$

$$(D_uD_vD_w + D_xD_yD_z)^n \left\{u^{-1}\; v^{\gamma-1+n}\; w^{-\beta}\; x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}}\; z^{\beta-\gamma}\right\}$$

(3.11)

$$= (n!)^2\,(\gamma)_n\,2^{-n}u^{-n-1}v^{\gamma-1}\,w^{\beta-n}\,x^{-\frac{n}{2}}y^{\frac{n}{2}-\frac{1}{2}}z^{\beta-\gamma}\,R_n\left(\beta;\gamma\,;\sqrt{\frac{xyz}{uvw}}\,\right)$$

$$(D_uD_vD_w + D_xD_yD_z)^n\left\{u^{-1}\;v^{-\alpha}\;w^{-\beta}\;x^{\frac{n}{2}}y^{\frac{n}{2}-\frac{1}{2}}\;z^{-1+\alpha+\beta+n}\right\}$$

(3.12)

$$= (-1)^n(n!)^2(\alpha+\beta)_n2^{-n}u^{-n-1}v^{-\alpha-n}\,w^{-\beta-n}\,x^{\frac{n}{2}}y^{\frac{n}{2}-\frac{1}{2}}z^{\alpha+\beta+n-1}\,G_n\left(\alpha;\beta\,;\sqrt{\frac{xyz}{uvw}}\,\right)$$

$$(D_uD_vD_wD_x + D_yD_z)^n\left\{u^{-1}\;v^{-1}\;w^{\nu-1+n}\;x^{-\nu}y^{\frac{n}{2}}\;z^{\frac{n}{2}-\frac{1}{2}}\right\}$$

(3.13)

$$= (n!)^2(\nu)_n2^{-n}u^{-1-\frac{1}{2}}v^{-n-1}\,w^{\nu-1}\,x^{-\nu-n}y^{\frac{n}{2}}z^{-\frac{n}{2}-\frac{1}{2}}\,R_{n,\nu}\left(\sqrt{\frac{xuvw}{yz}}\,\right)$$

$$\left(D_xD_{w_1}\cdots\;D_{w_s} - (-1)^{r+m}D_{y_1}\cdots\;D_{y_m}D_{z_1}\cdots\;D_{z_r}\right)^n$$

$$\times\;\left\{x^{-1}w_1^{\beta_1-1+n}\cdots\;w_s^{\beta_s-1+n}y_1^{\frac{n}{m}}\cdots\;y_m^{\frac{n-m+1}{m}}z_1^{-\alpha1}\cdots\;z_r^{-\alpha_r}\right\}$$

$$= (n!)(-1)^n(\beta_1)_n\cdots(\beta_s)_n\,x^{-n-1}w_1^{\beta_1-1}\cdots\,w_s^{\beta_s-1}y_1^{\frac{n}{m}}\cdots\,y_m^{\frac{n-(m-n}{m}}z_1^{-\alpha_1}\cdots\,z_r^{-\alpha_r}$$

(3.14) $$\times\;B_n^m\left[\alpha_1\cdots\alpha_s\,;\beta_1\cdots\beta_s\,;\;\frac{x\;w_1w_2\cdots\;w_s}{y_1y_2\cdots\;y_mz_1z_2\cdots\;z_r}\right]$$

$$(D_{x^m} + (-1)^mh(m^m)D_{y_1}\cdots\;D_{y_m})^n\left\{(x^{-m})^{-1}y_1^{\frac{n}{m}}\cdots\;y_m^{\frac{n-m+1}{m}}\right\}$$

(3.15)

$$= (-1)^n\,n!\,x^{mn+m-n}y_1^{\frac{n}{m}}\cdots\,y_m^{\frac{n-m+1}{m}}\,g_n^m(x\,y_1^{\frac{1}{m}}\cdots\cdots\,y_m^{\frac{1}{m}},h)$$

$$\left(D_{(vx)^m}+(-1)^m(-m^m)D_{y_1}\cdots\ D_{y_m}\right)^n\left\{[(vx)^{-m}]^{-1}y_1^{\frac{n}{m}}\cdots\ y_m^{\frac{n-m+1}{m}}\right\}$$

(3.16)

$$=(1)^n\ n!\ vx^{mn+m-n}y_1^{\frac{n}{m}}\cdots y_m^{\frac{nm+1}{m}}H_{m,n,\nu}(x\ y_1^{\frac{1}{m}}\cdots\cdots\ y_m^{\frac{1}{m}},h)$$

$$\left(D_{y_1}\cdot\cdot\ D_{y_k}-\left(\frac{-1}{2}\right)^k\ D_{z_1}\cdots\ D_{z_k}\right)^n$$

$$\times\left\{y_1^{\frac{\alpha+1}{k}-1+n}\cdots\ y_k^{\frac{\alpha+k}{k}-1+n}z_1^{-\frac{\alpha+\beta+n+1}{k}}\cdots\ z_k^{-\frac{\alpha+\beta+n+k}{k}}\right\}$$

(3.17)

$$=\frac{n!}{k^{k^n}}\ y_1^{\frac{\alpha+1}{k}-1}\cdots y_k^{\frac{\alpha+k}{k}-1}z_1^{-\frac{\alpha+\beta+n+1}{k}}\cdots z_k^{-\frac{\alpha+\beta+n+k}{k}}J_n^{(\alpha,\beta)}\left[1-2\left(\frac{y_1y_2\cdots\cdots\ y_k}{z_1z_2\cdots\cdots\ z_k}\right)^{\frac{1}{k}}\ ;k\right]$$

$$(D_uD_{v_1}\cdots\ D_{v_m}D_{w_1}\cdots\ D_{w_q}-(-1)^{m+r}D_{y_1}\cdots\ D_{y_m}D_{z_1}\cdots\ D_{z_r})^n$$

$$\times\left\{u^{-1}v_1^{\frac{\alpha+\beta\ n+1}{m}-1+n}\cdots\ v_m^{\frac{\alpha+\beta\ n+m}{m}-1+m}w_1^{b_1-1+n}\cdots\ w_q^{b_q-1+n}y_1^{\frac{n}{m}}\cdots\ y_m^{\frac{n+m-1}{m}}z_1^{-a_1}\cdots\ z_r^{-a_r}\right\}$$

$$=(-1)^n\ n!\frac{(\alpha+\beta\ n+1)_{mn}}{(1+\alpha+\beta\ n)_m\ m^{mn}}(b_1)_n\cdots(b_q)_n\ u^{-n-1}\ v_1^{\frac{\alpha+\beta\ n-m+1}{m}}\cdots v_m^{\frac{\alpha+\beta\ n)}{m}}$$

(3.18)

$$\times w_1^{b_1-1}\cdots w_q^{b_q-1}\ y_1^{\frac{n}{m}}\cdots y_m^{\frac{n+m-1}{m}}z_1^{-a_1}\cdots z_r^{-a_r}H_n^{(\alpha,\beta)}\left(\frac{u\ v_1v_2\cdots\cdots\ v_m\ w_1w_2\cdots\cdots\ w_q}{y_1y_2\cdots\cdots\ y_m\ z_1z_2\cdots\cdots\ z_r}\ ;m\right)$$

Some proofs are given:

Proof of (3.7):

$$(D_x+D_y)^n \left\{x^{-s-1}y^{-1}\right\}$$

$$= \sum_{r=0}^{n} \frac{(-n)_r(-1)^r}{r!} \frac{(1+s)_n(-1)^n}{(-s-n)_r} x^{-s-1-n+r}\ (1)_r(-1)^r y^{-1-r}$$

$$= (-1)^n\ n!\ x^{-s-1-n}y^{-1} \begin{pmatrix} s+n \\ n \end{pmatrix}\ {}_2F_1\left[\begin{matrix} -n, & 1; \\ -s-n; & \end{matrix}\ \frac{x}{y}\right].$$

$$= (-1)^n\ n!\ x^{-s-1-n}y^{-1} g_n^{(s)}\left(\frac{x}{y}\right)$$

Proof of (3.8):

$$(1+D_yD_z)^n \left\{y^{-x}e^{\frac{-1}{x}z}\right\}$$

$$= \sum_{r=0}^{n} \frac{(-n)_r(-1)^r}{r!} D_y^r \left\{y^{-x}\right\} D_z^r \left\{e^{\frac{-1}{x}z}\right\}$$

$$= n!\ y^{-x}e^{\frac{-1}{x}z}x^{-n}\frac{x^n}{n!}\ {}_2F_0\left[\begin{matrix} -n, & x; \\ & ; \end{matrix}\ \frac{-1}{xy}\right].$$

$$= n!\ y^{-x}e^{\frac{-1}{x}z}x^{-n}\varphi_n\left(\frac{y}{x}\right)$$

Proof of (3.9):

$$(1-D_x)^n \left\{x^{a-1+2n}\right\}$$

$$= \sum_{r=0}^{n} \frac{(-n)_r(-1)^{n-r}}{r!} \frac{(a+n)_n}{(a+n)_r} x^{a+n-1+r} \qquad \textit{Using } (3.6)$$

$$= n!\ \frac{(a)_2n}{(a)_n} x^{a-1+n}\ {}_1F_1\left[\begin{matrix} -n & ; \\ a+n; & \end{matrix}\ x\right].$$

$$= n! x^{a-1+n} R_n(a,x)$$

Proof of (3.10):

$$(D_u - D_x D_y)^n \left\{ u^{-1} x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} \right\}$$

$$= \sum_{r=0}^{n} \frac{(-n)_r}{r!} \frac{(-1)^n\ n!}{(-n)_r} u^{-n-1+r} \left(\frac{-n}{2}\right)_r (-1)^r x^{\frac{n}{2}-r} \left(\frac{-n}{2}+\frac{1}{2}\right)_r (-1)^r y^{\frac{n}{2}-\frac{1}{2}-r}$$

$$=(-1)^n\ n!\ u^{-n-1}\ x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}}\ {}_2F_0 \left[\begin{array}{c} -\frac{n}{2}\ ,\ \frac{-n}{2}+\frac{1}{2}\ ;\ \frac{u}{xy} \\ ; \end{array} \right]$$

$$=(-1)^n\ n!\ 2^{-n}\ u^{-n-1}\ x^{-\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}}\ H_n\left(\sqrt{\frac{xy}{u}}\right)$$

Proof of (3.11):

$$(D_u D_v D_w + D_x D_y D_z)^n \left\{ u^{-1}\ v^{\gamma-1+n}\ w^{-\beta}\ x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}}\ z^{\beta-\gamma} \right\}$$

$$= \sum_{r=0}^{n} \frac{(-n)_r (-1)^r}{r!} \frac{(-1)^n\ n!}{(-n)_r} u^{-n-1+r} \frac{(\gamma)_n}{(\gamma)_r} v^{\gamma-1+r} \frac{(\beta)_n\ (-1)^n}{(1-\beta-n)_r}$$

$$\times\ w^{-\beta-n+r} \left(\frac{-n}{2}\right)_r (-1)^r\ x^{\frac{n}{2}-r} \left(\frac{-n}{2}+\frac{1}{2}\right)_r (-1)^r\ y^{\frac{n}{2}-\frac{1}{2}-r} (\gamma-\beta)_r (-1)^r z^{\beta-\gamma-r}$$

$$= n!(\beta)_n(\gamma)_n\ u^{-n-1} v^{\gamma-1}\ w^{-\beta-n}\ x^{\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\beta-\gamma}$$

$$\sum_{r=0}^{n} \frac{(\frac{-n}{2})_r (\frac{-n}{2}+\frac{1}{2})_r (\gamma-\beta)_r}{r!(\gamma)_r (1-\beta-n)_r} \left(\frac{uvw}{xyz}\right)^r$$

$$= \frac{(n!)^2 (\gamma)_n}{2^n} u^{-n-1} v^{\gamma-1}\ w^{\beta-n}\ x^{-\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\beta-\gamma} \frac{(\beta)_n (2x)^n}{n!}\ {}_3$$

$$F_2 \left[\begin{array}{c} -\frac{n}{2}\ ,\ \frac{-n}{2}+\frac{1}{2}\ ,\gamma-\beta;\ \frac{uuw}{xyz} \\ \gamma,\ 1-\beta-n; \end{array} \right]$$

$$= (n!)^2\ (\gamma)_n\ 2^{-n} u^{-n-1} v^{\gamma-1}\ w^{\beta-n}\ x^{-\frac{n}{2}} y^{\frac{n}{2}-\frac{1}{2}} z^{\beta-\gamma}\ R_n\left(\beta;\gamma\ ;\sqrt{\frac{xyz}{uvw}}\ \right)$$

Proof of (3.16):

$$\left(D_{(vx)^m} + (-1)^m(-m^m)D_{y_1}\cdots\ D_{y_m}\right)^n \left\{[(vx)^{-m}]^{-1} y_1^{\frac{n}{m}} \cdots\ y_m^{\frac{n-m+1}{m}}\right\}$$

$$= \sum_{r=0}^{n} \frac{(-n)_r(-1)^r}{r!} \frac{(-n)_n}{(-n)_r} \left[(vx)^{-m}\right]^{-n-1+r} \left[(-1)^m\right]^r (-m)^r$$

$$\times \left(\frac{-n}{m}\right)_r (-1)^r y_1^{\frac{n}{m}-r} \cdots\cdots \left(\frac{-n+m-1}{m}\right)_r (-1)^r y_m^{\frac{n-m+1}{m}-r}$$

$$= (-1)^n n!\,(vx)^{mn+m}\, y_1^{\frac{n}{m}} \cdots\cdots\ y_m^{\frac{n-m+1}{m}}\ {}_mF_0\left[\begin{array}{l}\Delta(m,n);\\ -;\end{array}\ -\left(\frac{-m}{x\ y_1^{\frac{1}{m}} \cdots\cdots\ y_m^{\frac{1}{m}}}\right)^m\right]$$

$$= (1)^n\ n!\ vx^{mn+m-n} y_1^{\frac{n}{m}} \cdots\ y_m^{\frac{nm+1}{m}} H_{m,n,\nu}(x\ y_1^{\frac{1}{m}} \cdots\cdots\ y_m^{\frac{1}{m}}, h)$$

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