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Investigating the Use of Stratified Percentile Ranked Set Sampling Method for Estimating the Population Mean

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Abstract

Stratified percentile ranked set sampling (SPRSS) method is suggested for estimating the population mean. The SPRSS is compared with the simple random sampling (SRS), stratified simple random sampling (SSRS) and stratified ranked set sampling (SRSS). It is shown that SPRSS estimator is an unbiased estimator of the population mean of symmetric distributions and is more efficient than its counterparts using SRS, SSRS and SRSS based on the same number of measured units.

Keywords : *Simple random sampling; ranked set sampling; percentile ranked set sampling; efficiency; stratified ranked set sampling.*

1. Introduction

In last years, the ranked set sampling method, which was proposed by McIntyre (1952) to estimate mean pasture yields, was developed and modified by many authors to estimate the population parameters. Dell and Clutter (1972) showed that the mean of the RSS is an unbiased estimator of the population mean, whether or not there are errors in ranking. Al-Saleh and Al-Kadiri (2000) introduced double ranked set sampling for estimating the population mean. Al-Saleh and Al-Omari (2002) suggested multistage ranked set sampling that increase the efficiency of estimating the population mean for specific value of the sample size. Muttlak (2003b) suggested percentile ranked set sampling (PRSS) to estimate the population mean and showed that using PRSS procedure will reduce the errors in ranking comparing to RSS, since we only select and measure the pth or the qth percentile of the sample. Jemain and Al-Omari (2006) suggested double percentile ranked set sampling (DPRSS) for estimating the population mean and showed that the DPRSS mean is an unbiased estimator and more efficient than the SRS, RSS and PRSS if the underlying distribution is symmetric. Jemain and Al-Omari (2007) suggested multistage percentile ranked set sampling (MPRSS) to estimate the population mean, they showed that the efficiency of the mean estimator using MPRSS can be increased for specific value of the sample size by increasing the number of stages. For more details about RSS and its modifications see Al-Omari and Jaber (2008), Bouza (2002), Muttlak (2003a), Al-Nasser (2007) and Ohyama et al. (2008).

In this paper, stratified percentile ranked set sampling is suggested to estimate the population mean of symmetric and asymmetric distributions. This paper is organized as follows: In Section 2, some sampling methods are presented. Estimation of the population mean is given in Section 3. A simulation study is considered in Section 4. Finally, conclusions on the suggested estimator are given in Section 5

2. Sampling Methods

In stratified sampling method, the population of N units is divided into L non overlapping subpopulations each of $N_1, N_2, ..., N_L$ units, respectively, such that $N_1 + N_2 + ... + N_L = N$. These subpopulations are called strata. For full benefit from stratification, the size of the hth subpopulation, denoted by N_h for $h = 1, 2, ..., L$, must be known. Then the samples are drawn independently from each strata, producing samples sizes denoted by $n_1, n_2, ..., n_L$, such that the total sample size is $n = \sum_{h=1}^{L} n_h$. If a simple random sample is taken from each stratum, the whole procedure is known as stratified simple random sampling (SSRS).

The ranked set sampling (RSS) is suggested by McIntyre (1952) can be conducted by selecting n random samples from the population of size n units each, and ranking each unit within each set with respect to the variable of interest. Then an actual measurement is taken of the unit with the smallest rank from the first sample. From the second sample an actual measurement is taken from the second smallest rank, and the procedure is continued until the unit with the largest rank is chosen for actual measurement from the nth sample. Thus we obtain a total of n measured units, one from each ordered sample of size n and this completed one cycle. The cycle may be repeated m times to obtain a sample of size nm units.

The percentile ranked set sampling (PRSS) procedure is proposed by Muttlak (2003b). The PRSS can be described as: select n random samples each of size n units from the population and rank each sample with respect to a variable of interest. If the sample size n is even, select for measurement from the first $n/2$ samples the $p(n+1)$th smallest ranked unit and from the second $n/2$ samples the $q(n+1)$th smallest ranked unit where $0 \leq p \leq 1$ and $p+q=1$. If the sample size n is odd, select for measurement from the first $(n-1)/2$ samples the $p(n+1)$th smallest ranked unit and from the last $(n-1)/2$ samples the $q(n+1)$th smallest ranked unit, and the median from the middle sample. The cycle can be repeated m times if needed to get a sample of size nm units. Note that we will always take the nearest integer of $p(n+1)$th and $q(n+1)$th.

If the percentile ranked set sampling is used to select the sample units from each stratum, then the whole procedure is called a stratified percentile ranked set sampling (SPRSS). To illustrate the method, let us consider the following example for even sample size.

Example 1: Suppose that from the first stratum, we draw five samples each of 5 units, and from the second stratum, we draw seven samples each of seven units. These samples are ordered as given below.

Stratum 1: Five samples, each of size 5 ordered units as given below:

$$X_{11(1)}, X_{11(2)}, X_{11(3)}, X_{11(4)}, X_{11(5)}$$

$$X_{21(1)}, X_{21(2)}, X_{21(3)}, X_{21(4)}, X_{21(5)}$$

$$X_{31(1)}, X_{31(2)}, X_{31(3)}, X_{31(4)}, X_{31(5)}$$

$$X_{41(1)}, X_{41(2)}, X_{41(3)}, X_{41(4)}, X_{41(5)}$$

$$X_{51(1)}, X_{51(2)}, X_{51(3)}, X_{51(4)}, X_{51(5)}$$

Assume that $p = 37\%$ and $q = 63\%$. For $h = 1$, select $X_{ih(p(n_h+1))} = X_{i1(2)}$ for $i = 1, 2$, and select $X_{ih(q(n_h+1))} = X_{i1(4)}$ for $i = 4, 5$ and $X_{ih\left(\frac{n_1+1}{2}\right)} = X_{i1(3)}$ for $i = 3$, the following units are chosen from the first stratum

$$X_{11(2)}, X_{21(2)}, X_{31(3)}, X_{41(4)}, X_{51(4)}$$

Stratum 2: Seven samples, each with 7 ordered units are as given below

$$X_{12(1)}, X_{12(2)}, X_{12(3)}, X_{12(4)}, X_{12(5)}, X_{12(6)}, X_{12(7)}$$

$$X_{22(1)}, X_{22(2)}, X_{22(3)}, X_{22(4)}, X_{22(5)}, X_{22(6)}, X_{22(7)}$$

$$X_{32(1)}, X_{32(2)}, X_{32(3)}, X_{32(4)}, X_{32(5)}, X_{32(6)}, X_{32(7)}$$

$$X_{42(1)}, X_{42(2)}, X_{42(3)}, X_{42(4)}, X_{42(5)}, X_{42(6)}, X_{42(7)}$$

$$X_{52(1)}, X_{52(2)}, X_{52(3)}, X_{52(4)}, X_{52(5)}, X_{52(6)}, X_{52(7)}$$

$$X_{62(1)}, X_{62(2)}, X_{62(3)}, X_{62(4)}, X_{62(5)}, X_{62(6)}, X_{62(7)}$$

$$X_{72(1)}, X_{72(2)}, X_{72(3)}, X_{72(4)}, X_{72(5)}, X_{72(6)}, X_{72(7)}$$

For $h = 2$, select $X_{ih(p(n_h+1))} = X_{i1(3)}$ for $i = 1, 2, 3$, and select $X_{ih(q(n_h+1))} = X_{i1(5)}$ for $i = 5, 6, 7$ and $X_{ih\left(\frac{n_1+1}{2}\right)} = X_{i1(4)}$ for $i = 4$. Then the following units are chosen from the second stratum

$$X_{12(3)}, X_{22(3)}, X_{32(3)}, X_{42(4)}, X_{52(5)}, X_{62(5)}, X_{72(5)}.$$

Therefore, the selected SPRSS units are

$$X_{11(2)}, X_{21(2)}, X_{31(3)}, X_{41(4)}, X_{51(4)}, X_{12(3)}, X_{22(3)}, X_{32(3)}, X_{42(4)}, X_{52(5)}, X_{62(5)}, X_{72(5)}.$$

3. Estimation of the population mean

The SRS estimator of the population mean, μ, based on a sample of size n is given by

$$\overline{X}_{SRS} = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

with variance

$$Var\left(\overline{X}_{SRS}\right) = \frac{\sigma^2}{n}.$$

The estimator of the population mean for a RSS of size n is given by

$$\overline{X}_{RSS} = \frac{1}{n}\sum_{i=1}^{n} X_{i(i)},$$

with variance

$$Var\left(\overline{X}_{RSS}\right) = \frac{\sigma^2}{n} - \frac{1}{n^2}\sum_{i=1}^{n}\left(\mu_{(i)} - \mu\right)^2,$$

where $\mu_{(i)}$ is the mean of the ith order statistics, $X_{(i)}$ of a sample of size n.

In the case of stratified percentile ranked set sampling (SPRSS), when n_h is even, the estimator of the population mean is defined as

$$\overline{X}_{SPRSS1} = \sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}} X_{ih(p(n_h+1))} + \sum_{i=\frac{n_h}{2}+1}^{n} X_{ih(q(n_h+1))}\right), \tag{3.1}$$

where $W_h = \frac{N_h}{N}$, N_h is the stratum size, N is the total population size and $p + q = 1$.

The variance of SPRSS1 is given by

$$\begin{aligned}
Var\left(\overline{X}_{SPRSS1}\right) &= Var\left[\sum_{h=1}^{L}\frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h}{2}} X_{ih(p(n_h+1))} + \sum_{i=\frac{n_h}{2}+1}^{n} X_{ih(q(n_h+1))}\right)\right] \\
&= \sum_{h=1}^{L}\frac{W_h^2}{n_h^2}\left(\sum_{i=1}^{\frac{n_h}{2}} Var\left(X_{ih(p(n_h+1))}\right) + \sum_{i=\frac{n_h}{2}+1}^{n} Var\left(X_{ih(q(n_h+1))}\right)\right) \\
&= \sum_{h=1}^{L}\frac{W_h^2}{n_h^2}\left(\sum_{i=1}^{\frac{n_h}{2}} Var\left(X_{ih(p(n_h+1))}\right) + \sum_{i=\frac{n_h}{2}+1}^{n} Var\left(X_{ih(q(n_h+1))}\right)\right)
\end{aligned}$$

$$= \sum_{h=1}^{L} \frac{W_h^2}{n_h^2} \left(\sum_{i=1}^{\frac{n_h}{2}} \sigma^2_{ih(p)} + \sum_{i=\frac{n_h}{2}+1}^{n} \sigma^2_{ih(q)} \right). \tag{3.2}$$

When n_h is odd, the SPRSS estimator of the population mean is given by

$$\overline{X}_{SPRSS2} = \sum_{h=1}^{L} \frac{W_h}{n_h} \left(\sum_{i=1}^{\frac{n_h-1}{2}} X_{ih(p(n_h+1))} + \sum_{i=\frac{n_h-1}{2}+2}^{n_h} X_{ih(q(n_h+1))} + X_{ih\left(\frac{n_h+1}{2}\right)} \right) \tag{3.3}$$

and the variance of SPRSS2 is

$$Var\left(\overline{X}_{SPRSS2}\right) =$$
$$Var\left[\sum_{h=1}^{L} \frac{W_h}{n_h} \left(\sum_{i=1}^{\frac{n_h-1}{2}} X_{ih(p(n_h+1))} + \sum_{i=\frac{n_h-1}{2}+2}^{n_h} X_{ih(q(n_h+1))} + X_{ih\left(\frac{n_h+1}{2}\right)} \right) \right]$$
$$= \sum_{h=1}^{L} \frac{W_h^2}{n_h^2} \left(\sum_{i=1}^{\frac{n_h-1}{2}} Var\left(X_{ih(p(n_h+1))}\right) + \sum_{i=\frac{n_h-1}{2}+2}^{n_h} Var\left(X_{ih(q(n_h+1))}\right) \right.$$
$$\left. + Var\left(X_{ih\left(\frac{n_h+1}{2}\right)}\right) \right)$$

$$= \sum_{h=1}^{L} \frac{W_h^2}{n_h^2} \left(\sum_{i=1}^{\frac{n_h-1}{2}} \sigma^2_{ih(p)} + \sum_{i=\frac{n_h-1}{2}+2}^{n_h} \sigma^2_{ih(q)} + \sigma^2_{ih(q2)} \right) \tag{3.4}$$

Property 1: If the underlying distribution is symmetric about μ, $ih(q2)$ then $E\left(\overline{X}_{SPRSS1}\right) = \mu$ and $E\left(\overline{X}_{SPRSS2}\right) = \mu$.
Proof: If n_h is even, we have

$$E\left(\overline{X}_{SPRSS2}\right) = E\left[\sum_{h=1}^{L} \frac{W_h}{n_h} \left(\sum_{i=1}^{\frac{n_h}{2}} X_{ih(p(n_h+1))} + \sum_{i=\frac{n_h}{2}+1}^{n} X_{ih(q(n_h+1))} \right) \right]$$
$$= \sum_{h=1}^{L} \frac{W_h}{n_h} \left(\sum_{i=1}^{\frac{n_h}{2}} E\left(X_{ih(p(n_h+1))}\right) + \sum_{i=\frac{n_h}{2}+1}^{n} E\left(X_{ih(q(n_h+1))}\right) \right)$$
$$= \sum_{h=1}^{L} \frac{W_h}{n_h} \left(\sum_{i=1}^{\frac{n_h}{2}} \mu_{h(p)} + \sum_{i=\frac{n_h}{2}+1}^{n} \mu_{h(q)} \right),$$

where $\mu_{h(p)}$ and $\mu_{h(q)}$ are the means of the order statistics which correspond to the pth and qth percentiles, respectively. Since the distribution is symmetric about μ, then $\mu_{h(p)} + \mu_{h(q)} = 2\mu$. Therefore, we have

$$\begin{aligned} E\left(\overline{X}_{SPRSS1}\right) &= \sum_{h=1}^{L} \tfrac{W_h}{n_h}\left(\tfrac{n_h}{2}\mu_{h(p)} + \tfrac{n_h}{2}\mu_{h(q)}\right) \\ &= \sum_{h=1}^{L} \tfrac{W_h}{n_h}\left(\tfrac{n_h}{2}\left(\mu_{h(p)} + \mu_{h(q)}\right)\right) \\ &= \sum_{h=1}^{L} \tfrac{W_h}{n_h}\left(\tfrac{n_h}{2}\left(2\mu_h\right)\right) \\ &= \sum_{h=1}^{L} W_h\mu_h = \mu. \end{aligned}$$

If n_h is odd, then

$$\begin{aligned} &E\left(\overline{X}_{SPRSS2}\right) = \\ &E\left[\sum_{h=1}^{L} \frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}} X_{ih(p(n_h+1))} + \sum_{i=\frac{n_h-1}{2}+2}^{n_h} X_{ih(q(n_h+1))} + X_{ih\left(\frac{n_h+1}{2}\right)}\right)\right] \\ &= \sum_{h=1}^{L} \frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}} E\left(X_{ih(p(n_h+1))}\right) + \sum_{i=\frac{n_h-1}{2}+2}^{n_h} E\left(X_{ih(q(n_h+1))}\right) + E\left(X_{ih\left(\frac{n_h+1}{2}\right)}\right)\right) \\ &= \sum_{h=1}^{L} \frac{W_h}{n_h}\left(\sum_{i=1}^{\frac{n_h-1}{2}} \mu_{h(p)} + \sum_{i=\frac{n_h-1}{2}+2}^{n_h} \mu_{h(q)} + \mu_{h(q_2)}\right), \end{aligned}$$

where $\mu_{h(p)}$ is the mean for the pth percentile in the first $\left(\frac{n_h-1}{2}\right)$ samples in stratum h. $\mu_{h(q)}$ is the mean for the qth percentile in the last $\left(\frac{n_h-1}{2}\right)$ samples in stratum h. μ_h is the mean for the stratum h. Since the distribution is symmetric about μ, then we have $\mu_{h(p)} + \mu_{h(p)} = 2\mu_h$. Therefore,

$$\begin{aligned}
E\left(\overline{X}_{SPRSS2}\right) &= \sum_{h=1}^{L} \frac{W_h}{n_h} \left[\left(\frac{n_h-1}{2}\right)\mu_{h(p)} + \left(\frac{n_h-1}{2}\right)\mu_{h(q)} + \mu_h\right] \\
&= \sum_{h=1}^{L} \frac{W_h}{n_h} \left[\left(\frac{n_h-1}{2}\right)\left(\mu_{h(p)} + \mu_{h(q)}\right) + \mu_h\right] \\
&= \sum_{h=1}^{L} \frac{W_h}{n_h} \left[\left(\frac{n_h-1}{2}\right) 2\mu_h + \mu_h\right] \\
&= \sum_{h=1}^{L} \frac{W_h}{n_h} \left((n_h-1)\mu_h + \mu_h\right) \\
&= \sum_{h=1}^{L} \frac{W_h}{n_h} \left(n_h \mu_h\right) \\
&= \sum_{h=1}^{L} W_h \mu_h = \mu.
\end{aligned}$$

Property 2: If the distribution is symmetric about μ, then

$$Var\left(\overline{X}_{SPRSS1}\right) < Var\left(\overline{X}_{SRS}\right)$$

and $Var\left(\overline{X}_{SPRSS2}\right) < Var\left(\overline{X}_{SRS}\right)$..

Proof: If the sample size is even, the variance of

$$\overline{X}_{SPRSS1}$$

is given by $Var\left(\overline{X}_{SPRSS1}\right) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\sigma_{ih(q)}^2$.

But $\sigma_{ih(q)}^2 < \sigma_h^2$ for each stratum $h = 1, 2, \cdots, L$, which implies that

$$Var\left(\overline{X}_{SPRSS1}\right) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\sigma_{ih(q)}^2 < \sum_{h=1}^{L} \frac{W_h^2}{n_h}\sigma_h^2 = Var(_{SSRS}) < Var(\overline{X}_{SRS}).$$

The proof is the same for odd sample size.

4. Simulation study

In this section, a simulation study is designed for symmetric and asymmetric distributions with samples of sizes $n = 7, 12, 14, 15, 18$ to compare the SPRSS with the SRS, SSRS and SRSS methods. Without loss of generality, we assumed that the population is partitioned into two or three strata. Using 100000 replications, estimates of the means, variances and mean square errors are computed. The efficiency of SPRSS relative to SRS, SSRS and SRSS when the parent distribution is symmetric is given by

$$eff\left(\overline{X}_{SPRSS}, \overline{X}_{M}\right) = \frac{Var\left(\overline{X}_{M}\right)}{Var\left(\overline{X}_{SPRSS}\right)},$$

$M = SRS, SSRS, SRSS$, and when the distribution is asymmetric

$$eff\left(\overline{X}_{SPRSS}, \overline{X}_{M}\right) = \frac{MSE\left(\overline{X}_{M}\right)}{MSE\left(\overline{X}_{SPRSS}\right)},$$

where MSE is the mean square error.

In Table 1 to Table 6 we summarized the efficiency values of the estimators, while in Table 7 the bias values of the estimators for the mean of asymmetric distributions considered in this study are presented.

Table 1: The efficiency of SPRSS relative to SRSS, SSRS and SRS for $n = 14$ and samples of sizes $n_1 = 8$ and $n_2 = 6$

	P	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	20%	1.1828	1.1517	1.1345
	30%	1.5860	2.0877	2.0526
	40%	3.2800	3.4400	3.3600
Normal (0,1)	20%	1.5208	1.5578	1.6336
	30%	3.0725	2.3023	2.2773
	40%	4.7656	5.6797	5.5781
Student T (3)	20%	2.1553	2.5678	2.4287
	30%	3.7014	3.8071	3.8232
	40%	6.5019	8.2113	7.9245
Geometric (0.5)	20%	1.0058	1.1247	1.3992
	30%	2.8384	2.9024	2.8013
	40%	3.2565	3.5043	3.4630
Beta (5,2)	20%	1.3750	1.1875	1.1250
	30%	1.8000	1.9000	1.8000
	40%	3.5268	3.2314	3.1140
Weibull (1,2)	20%	1.7381	1.8571	1.8214
	30%	2.5345	2.9897	2.6379
	40%	4.1786	4.5714	4.5000

Table 2: The efficiency of SPRSS relative to SRSS, SSRS and SRS for $n = 7$ and samples sizes $n_1 = 4$ and $n_2 = 3$

	P	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	20%	1.3440	1.8680	1.8520
	30%	2.1044	2.0097	2.1750
	40%	2.1137	2.2431	2.3345
Normal (0,1)	20%	1.9521	1.2206	1.2100
	30%	2.3041	1.9804	1.8232
	40%	2.9172	3.3480	3.2344
Student T (3)	20%	2.8528	3.0891	3.0697
	30%	3.5498	4.3955	4.3092
	40%	4.3260	5.9241	5.2762
Geometric (0.5)	20%	2.6179	2.5990	2.5437
	30%	3.0745	3.0711	3.1328
	40%	3.0875	3.0725	3.1175
Beta (5,2)	20%	1.1394	1.0606	1.0905
	30%	1.9593	2.1604	2.0714
	40%	2.5636	2.6636	2.5647
Weibull (1,2)	20%	1.1090	1.0081	1.0968
	30%	1.5931	1.3085	1.4853
	40%	1.7891	1.6922	1.7074

Table 3: The efficiency of SPRSS relative to SRSS, SSRS and SRS for $n = 12$ and samples sizes $n_1 = 5$ and $n_2 = 7$

	P	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	20%	1.8451	1.7961	1.7765
	30%	2.5200	2.9800	2.7800
	40%	4.9913	2.2174	4.1080
Normal (0,1)	20%	2.6335	3.7738	3.7538
	30%	3.9541	4.8349	4.6514
	40%	4.2795	5.6818	5.2101
Student T (3)	20%	3.2145	3.2300	3.1196
	30%	4.1058	3.4658	3.3209
	40%	5.7732	4.1486	4.2331
Geometric (0.5)	20%	1.0586	1.5624	1.5447
	30%	2.0228	2.5095	2.3308
	40%	3.4332	3.9312	3.7409
Beta (5,2)	20%	1.6333	1.4333	1.4000
	30%	2.7692	2.6923	2.6154
	40%	2.9960	3.1456	3.2208
Weibull (1,2)	20%	1.2806	1.6706	1.8483
	30%	2.0693	2.5218	2.3723
	40%	3.6200	2.9800	2.8837

Table 4: The efficiency of SPRSS relative to SRSS, SSRS and SRS for $n = 18$ and samples sizes $n_1 = 4$, $n_2 = 6$ and $n_3 = 8$.

	P	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	20%	1.8947	2.5789	2.4211
	30%	1.9950	3.5625	3.6750
	40%	2.1693	4.5613	4.3474
Normal (0,1)	20%	2.2571	3.6074	3.4049
	30%	3.4083	5.4793	5.2840
	40%	4.8214	7.5000	6.9107
Student T (3)	20%	2.8412	2.9687	2.9411
	30%	3.5000	4.5377	4.8115
	40%	7.4014	8.7132	8.1366
Geometric (0.5)	20%	1.8593	2.0281	2.1414
	30%	2.6932	3.2260	3.0325
	40%	2.7103	3.4273	3.3021
Beta (5,2)	20%	1.1927	1.0859	2.0601
	30%	2.1643	3.1298	3.6878
	40%	2.8471	3.3330	3.7958
Weibull (1,2)	20%	1.6200	2.5200	2.3800
	30%	2.2300	3.3400	3.1600
	40%	2.9455	3.4873	3.1951

Table 5: The efficiency of SPRSS relative to SRSS, SSRS and SRS for $n = 15$ and samples sizes $n_1 = 3$, $n_2 = 5$ and $n_3 = 7$.

	P	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	20%	1.0909	2.7273	2.5455
	30%	2.2000	3.5000	3.8000
	40%	3.9091	5.4545	4.9909
Normal (0,1)	20%	1.1930	3.6349	3.5977
	30%	2.3750	5.1750	5.2500
	40%	2.9684	5.8342	5.4632
Student T (3)	20%	2.1549	2.9970	2.8256
	30%	3.1709	4.1709	4.1966
	40%	4.0198	6.2099	5.9842
Geometric (0.5)	20%	1.4494	2.1685	2.0112
	30%	3.1542	6.3391	5.8637
	40%	3.5786	6.4786	5.9429
Beta (5,2)	20%	1.0709	2.3134	2.1849
	30%	1.7906	3.9138	3.7497
	40%	2.7355	4.1160	4.2207
Weibull (1,2)	20%	1.7647	2.5294	2.2059
	30%	2.9000	4.6222	4.3778
	40%	3.9500	5.1333	5.1167

Table 6: The efficiency of SPRSS relative to SRSS, SSRS and SRS for $n = 18$ and samples sizes $n_1 = 10$ and $n_2 = 8$.

	P	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRSS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SSRS}\right)$	$eff\left(\overline{X}_{SPRSS}, \overline{X}_{SRS}\right)$
Uniform (0,1)	20%	1.0652	1.0217	1.0000
	30%	1.5000	2.6786	2.6429
	40%	3.2500	3.3500	3.3000
Normal (0,1)	20%	1.0668	1.3886	1.3762
	30%	2.6777	3.4239	3.4112
	40%	4.8698	5.8425	5.8082
Student T (3)	20%	2.7574	2.7969	2.8507
	30%	3.4524	4.4960	4.6905
	40%	7.5476	9.4881	8.0714
Geometric (0.5)	20%	1.0284	1.5183	1.5007
	30%	2.5899	2.1304	2.0362
	40%	2.6512	3.3393	3.3006
Beta (5,2)	20%	1.1429	1.0000	1.9143
	30%	2.1214	1.7476	2.2697
	40%	2.5556	3.4361	3.5014
Weibull (1,2)	20%	1.2212	1.0708	1.0531
	30%	2.2336	2.1308	2.1121
	40%	2.5250	3.0250	4.3750

Table 7: The bias values of the estimators for different strata and sample sizes

		SPRSS		SRSS	SSRS	SRS
	P = 20%	P = 30%	P = 40%			
	when $n = 14$, with two strata $n_1 = 8$ and $n_2 = 6$					
Geometric (0.5)	0.0474	0.0139	0.0135	0.0640	0.0727	0.0716
Beta (5,2)	0.0392	0.0337	0.0351	0.0360	0.0371	0.0365
Weibull (1,2)	0.0453	0.0473	0.0484	0.0475	0.0572	0.0562
	when $n = 7$, with two strata $n_1 = 4$ and $n_2 = 3$					
Geometric (0.5)	0.0141	0.0109	0.0144	0.0171	0.1453	0.1422
Beta (5,2)	0.0742	0.0774	0.0524	0.0772	0.0743	0.0728
Weibull (1,2)	0.0772	0.0699	0.0621	0.0970	0.1142	0.1120
	when $n = 12$, with two strata $n_1 = 5$ and $n_2 = 7$					
Geometric (0.5)	0.0209	0.0166	0.0153	0.0421	0.0852	0.0831
Beta (5,2)	0.0386	0.0403	0.0408	0.0410	0.0436	0.0426
Weibull (1,2)	0.0540	0.0512	0.0565	0.0594	0.0671	0.0654
	when $n = 18$, with three strata $n_1 = 4$, $n_2 = 6$ and $n_3 = 8$					
Geometric (0.5)	0.0077	0.0038	0.0030	0.0067	0.0590	0.0555
Beta (5,2)	0.0095	0.0096	0.0099	0.0102	0.0300	0.0284
Weibull (1,2)	0.0121	0.0113	0.0110	0.0152	0.0462	0.0436
	when $n = 15$, with three strata $n_1 = 3$, $n_2 = 5$ and $n_3 = 7$					
Geometric (0.5)	0.0091	0.0079	0.0067	0.0109	0.0720	0.0668
Beta (5,2)	0.0094	0.0122	0.0104	0.0110	0.0367	0.0340
Weibull (1,2)	0.0141	0.0117	0.0109	0.0160	0.0565	0.0524
	when $n = 18$, with two strata $n_1 = 10$ and $n_2 = 8$					
Geometric (0.5)	0.0139	0.0045	0.0043	0.0201	0.0560	0.0555
Beta (5,2)	0.0273	0.0292	0.0259	0.0294	0.0287	0.0062
Weibull (1,2)	0.0393	0.0385	0.0375	0.0434	0.0441	0.0437

5. Conclusions

In this paper, a new estimator of the population mean using SPRSS is suggested. The performance of the estimator based on SPRSS is compared with those found using SRSS, SSRS and SRS for the same number of measured units. It is found that SPRSS produces an unbiased estimator of the population mean and it is more efficient than SRSS, SSRS and SRS. Thus, the SPRSS should be more preferred than SRSS, SSRS and SRS for both symmetric and asymmetric distributions.

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