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3-difference cordiality of some corona graphs

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Abstract

Let G be a (p,q) graph. Let $f : V(G) \rightarrow \{1,2,\ldots,k\}$ be a map where k is an integer $2 \leq k \leq p$. For each edge uv, assign the label $|f(u) - f(v)|$. f is called k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph. In this paper we investigate 3-difference cordial labeling behavior of $T_n \odot K_1$, $T_n \odot 2K_1$, $T_n \odot K_2$, $A(T_n) \odot K_1$, $A(T_n) \odot 2K_1$, $A(T_n) \odot K_2$.

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1. Introduction

Let G be a (p, q) simple, undirected, finite graph on p vertices and q edges. Graph labeling is used in several areas of science and technology like coding theory, astronomy, circuit design etc. For more details refer Gallian [1]. In 1987, Cahit [1] introduced the cordial labeling of graphs. Let f be a function from the vertices of G to $\{0, 1\}$ and for each edge xy assign the label $|f(x) - f(y)|$. Call f a *cordial labeling* of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1, and the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. Let G_1, G_2 respectively be (p_1, q_1), (p_2, q_2) graphs. The corona of G_1 with G_2, $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. The notion of difference cordial labeling was introduced by R. Ponraj, S. Sathish Narayanan and R. Kala in [4]. Seoud and Salman [10], studied the difference cordial labeling behavior of some families of graphs and they are ladder, triangular ladder, grid, step ladder and two sided step ladder graphs etc. Recently Ponraj et al. [4], introduced the concept of k-difference cordial labeling of graphs and studied the 3-difference cordial labeling behavior of star, m copies of star etc. In [5, 6, 7, 8, 9] they discussed the 3-difference cordial labeling behavior of path, cycle, complete graph, complete bipartite graph, star, bistar, comb, double comb, quadrilateral snake, wheel, helms, flower graph, sunflower graph, lotus inside a circle, closed helm, double wheel, union of graphs with the star, union of graphs with splitting graph of star, union of graphs with subdivided star, union of graphs with bistar, $P_n \cup P_n$, $(C_n \odot K_1) \cup (C_n \odot K_1)$, $F_n \cup F_n$, $K_{1,n} \odot K_2$, $P_n \odot 3K_1$, mC_4, $spl(K_{1,n})$, $DS(B_{n,n})$, $C_n \odot K_2$, $C_4^{(t)}$, $S(K_{1,n})$, $S(B_{n,n})$, $DA(T_n) \odot K_1$, $DA(T_n) \odot 2K_1$, $DA(T_n) \odot K_2$, $DA(Q_n) \odot K_1$, $DA(Q_n) \odot 2K_1$, triangular snake, alternate triangular snake, alternate quadrilateral snake, irregular triangular snake, irregular quadrilateral snake, double triangular snake, double quadrilaterla snake, double alternate triangular snake, and double alternate quadrilateral snake and some more graphs. In this paper we investigate 3-difference cordial labeling behavior of $T_n \odot K_1$, $T_n \odot 2K_1$, $T_n \odot K_2$, $A(T_n) \odot K_1$, $A(T_n) \odot 2K_1$, $A(T_n) \odot K_2$. Terms are not defined here follows from Harary [2].

2. k-Difference cordial labeling

Definition 2.1. *Let G be a (p, q) graph. Let $f : V(G) \to \{1, 2, \dots, k\}$ be a map. For each edge uv, assign the label $|f(u) - f(v)|$. f is called a k-difference cordial labeling of G if $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$ where $v_f(x)$ denotes the number of vertices labelled with x, $e_f(1)$ and $e_f(0)$ respectively denote the number of edges labelled with 1 and not labelled with 1. A graph with a k-difference cordial labeling is called a k-difference cordial graph.*

Example 2.1. *An example of 3-difference cordial labeling is given in figure 1.*

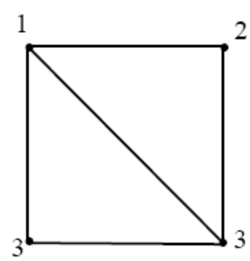

FIGURE 1

We investigate the 3-difference cordial labeling behavior of some path related graphs. The triangular snake T_n is obtained from the path P_n by replacing each edge of the path by a triangle C_3. Let P_n be the path $u_1 u_2 \dots u_n$. Let $V(T_n) = V(P_n) \cup \{v_i : 1 \leq i \leq n-1\}$ and $E(T_n) = E(P_n) \cup \{u_i v_i, v_i u_{i+1} : 1 \leq i \leq n-1\}$. We now investigate the 3-difference cordiality of corona of triangular snakes T_n with K_1, $2K_1$ and K_2.

Theorem 2.1. *If $n \leq 13$ then $T_n \odot K_1$ is 3-difference cordial.*

Proof. Clearly, $T_n \odot K_1$ has 4n-2 vertices 5n-4 edges. Let $V(T_n \odot K_1) = V(T_n) \cup \{w_i : 1 \leq i \leq n\} \cup \{z_i : 1 \leq i \leq n-1\}$ and $E(T_n \odot K_1) = E(T_n) \cup \{u_i w_i : 1 \leq i \leq n\} \cup \{v_i z_i : 1 \leq i \leq n-1\}$. Consider the path vertices u_i. Assign the label 1 to the path vertices $u_1, u_2, u_3, ...$ Then assign the label 2 to all the vertices v_i $(1 \leq i \leq n-1)$. Assign the label 3 to all the vertices of w_i $(1 \leq i \leq n)$. Next we move to the vertices w_i. Assign the label 3 and 2 to the vertices w_6 and w_7 respectively. Then assign the label 1 to the vertices z_{7i+1}, z_{7i+5} for i=0,1. Assign the label 2 to the vertices z_{7i+2}, z_{7i+4} for i=0,1. For the values of i=0,1 assign the label 3 to the vertices z_{7i+3}. The vertex and edge conditions are given in table 2.1 and table 2.2.

Nature of n	$e_f(0)$	$e_f(1)$
n=3,5	$\frac{5n-3}{2}$	$\frac{5n-5}{2}$
n=4,6,8,10,12	$\frac{5n-4}{2}$	$\frac{5n-4}{2}$
n=7,9,11,13	$\frac{5n-5}{2}$	$\frac{5n-3}{2}$

Table 2.1:

values of n	$v_f(1)$	$v_f(2)$	$v_f(3)$
n=5,8,11	$\frac{4n-2}{3}$	$\frac{4n-2}{3}$	$\frac{4n-2}{3}$
n=3,6,9	$\frac{4n}{3}$	$\frac{4n-3}{3}$	$\frac{4n-3}{3}$
n=4,7	$\frac{4n-1}{3}$	$\frac{4n-4}{3}$	$\frac{4n-1}{3}$
n=10,13	$\frac{4n-1}{3}$	$\frac{4n-1}{3}$	$\frac{4n-4}{3}$
n=12	$\frac{4n-3}{3}$	$\frac{4n}{3}$	$\frac{4n-3}{3}$

Table 2.2:

□

Example 2.2. *A 3-difference cordial labeling of $T_6 \odot K_1$ is given in figure 2.*

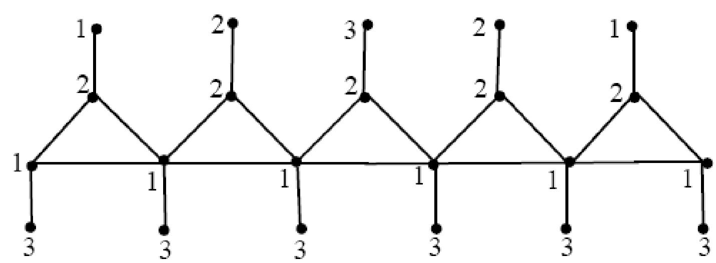

FIGURE 2

Theorem 2.2. *$T_n \odot 2K_1$ is 3-difference cordial for all values of n .*

Proof. Clearly the order an size of $T_n \odot 2K_1$ are 6n-3 and 7n-5 respectively. Let $V(T_n \odot 2K_1) = V(T_n) \cup \{w_i, w'_i : 1 \le i \le n\} \cup \{z_i, z'_i : 1 \le i \le n-1\}$ and $E(T_n \odot 2K_1) = E(T_n) \cup \{u_i w_i, u_i w'_i : 1 \le i \le n\} \cup \{v_i z_i, v_i z'_i : 1 \le i \le n-1\}$. Consider the path vertices u_i as in the label 2 to the vertices $u_1, u_3, u_5, ...$ Assign the label 3 to the vertices $u_2, u_4, u_6, ...$ We now consider the vertices v_i. Assign the label 2 to the vertices v_1. Then assign the label 1 to the vertices $v_2, v_3, v_4, ...$ Next we move to the vertices z_i and z'_i. Assign the label 1 and 4 to all the vertices of z_i $(1 \le i \le n-1)$ and z'_i $(1 \le i \le n-1)$ respectively. Consider the vertices w_i and w'_i. Assign the labels 1,1 to the vertices w_1, w_2 respectively and assign the labels 2,3 to the vertices w'_1, w'_2 respectively. Now we assign the label 2 to the vertices $w_3, w_4, w_5, ...$ Then assign the label 3 to the vertices $w'_3, w'_5, w'_7, ...$ Finally assign the label 2 to the vertices $w'_4, w'_6, w'_8, ...$ The vertex and edge condition is given by $v_f(1) = v_f(2) = v_f(3) = 2n-1$.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 1 \pmod 2$	$\frac{7n-5}{2}$	$\frac{7n-5}{2}$
$n \equiv 0 \pmod 2$	$\frac{7n-6}{2}$	$\frac{7n-4}{2}$

Table 2.3:

□

Theorem 2.3. *$T_n \odot K_2$ is 3-difference cordial for all values of n .*

Proof. Clearly the order an size of $T_n \odot K_2$ are 6n-3 and 7n-5 respectively. Let $V(T_n \odot K_2) = V(T_n) \cup \{w_i, w'_i : 1 \le i \le n\} \cup \{z_i, z'_i : 1 \le i \le n-1\}$ and $E(T_n \odot K_2) = E(T_n) \cup \{u_i w_i, u_i w'_i, w_i w'_i : 1 \le i \le n\} \cup \{v_i z_i, v_i z'_i, z_i z'_i : 1 \le i \le n-1\}$.

Case 1. $n \equiv 1 \pmod 4$. First we consider the path vertices u_i. Assign the label 1 to the all path vertices u_i $(1 \le i \le n)$. Then we move to the vertices v_i. Assign the label 1,1,1,2 to the first four vertices v_1, v_2, v_3, v_4 respectively. Then we assign the label 1,1,1,2 to the next four vertices v_5, v_6, v_7, v_8 respectively. Continue this process we assign the next four vertices and so on. Note that in this case th last vertex received the label 2. Consider the vertices w_i and w'_i. Assign the lbel 2 to all the vertices w_i $(1 \le i \le n)$ and assign the label 3 to all the vertices of w'_i $(1 \le i \le n)$. Next we move to the vertices z_i and z_i'. Assign the label 2,2,2,1 to the first four vertices z_1, z_2, z_3, z_4 respectively. Then we assign the labels 2,2,2,1 to the next four vertices z_5, z_6, z_7, z_8 respectively. Proceeding like this we assign the label to the next four vertices and so on. Clearly the last vertex z_{n-1} received the label 1. Assign the label 3 to all the vertices of z_i $(1 \leq i \leq n-1)$. The vertex and edge condition is given by $v_f(1) = v_f(2) = v_f(3) = 2n-1$ and $e_f(0) = \frac{9n-7}{2}$ and $e_f(1) = \frac{9n-5}{2}$.

Case 2. $n \equiv 2 \pmod 4$. Assign the label to the vertices u_i, w_i and w_i' $(1 \leq i \leq n)$ as in case 1. Consider the vertices v_i. Assign the label 1 to the vertex v_1. Now we assign the labels 1,1,1,2 to the vertices v_2, v_3, v_4, v_5 respectively. Then we assign the labels 1,1,1,2 to the next four vertices v_6, v_7, v_8, v_9 respectively. Continuing like this assign the label to the next four vertices and so on. Note that in this case the last vertex v_{n-1} received the label 2. Now we move to the vertices z_i and z_i'. Assign the label to the vertices z_i' $(1 \leq i \leq n-1)$ as in case 1. Then we assign the label 2 to the vertex z_1. Now we assign the labels 2,2,2,1 to the four vertices z_2, z_3, z_4, z_5 respectively. Then assign the labels 2,2,2,1 to the next four vertices z_6, z_7, z_8, z_9 respectively. Proceeding like this assign the label to the next four vertices and so on. Clearly the last vertex z_{n-1}' received the label 1. The vertex and edge condition of this case is $v_f(1) = v_f(2) = v_f(3) = 2n-1$ and $e_f(0) = e_f(1) = \frac{9n-6}{2}$.

Case 3. $n \equiv 3 \pmod 4$. Consider the path vertices u_i. Assign the label to the vertices u_i $(1 \leq i \leq n)$ as in case 1. Next we assign the labels 1,1 to the vertices v_1, v_2 respectively. Assign the labels 1,1,1,2 to the four vertices v_3, v_4, v_5, v_6 respectively. Then we assign the labels 1,1,1,2 to the next four vertices v_7, v_8, v_9, v_{10} respectively. Continuing like this assign the label to the next four vertices and so on. Note that in this case the last vertex v_{n-1} received the label 2. Assign the label to the vertices w_i $(1 \leq i \leq n)$ and w_i' $(1 \leq i \leq n)$ as in case 1. Now we move to the vertices z_i and z_i'. Assign the labels 2,2 to the vertices z_1, z_2 respectively. Next we assign the label 2,2,2,1 to the vertices z_3, z_4, z_5, z_6 respectively. Then we assign the label 2,2,2,1 to the vertices z_7, z_8, z_9, z_{10} respectively. Proceeding like this assign the label to the next four vertices and so on. Clearly the last vertex z_{n-1} received the label 1. Assign the label z_i' as in case 1. The vertex and edge condition is $v_f(1) = v_f(2) = v_f(3) = 2n-1$ and $e_f(0) = \frac{9n-5}{2}$ and $e_f(1) = \frac{9n-7}{2}$. □

Example 2.3. *A 3-difference cordial labeling of $T_5 \odot K_2$ is given in figure 3.*

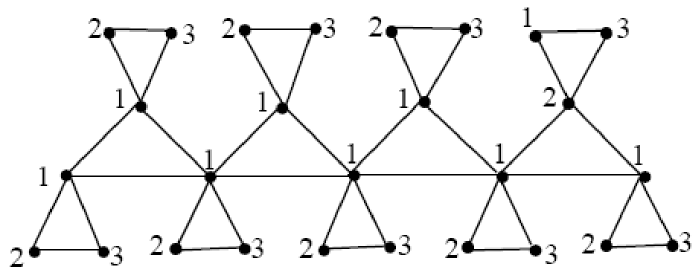

FIGURE 3

An alternate triangular snake $A(T_n)$ is obtained from a path $u_1u_2...u_n$ by joining u_i and u_{i+1} (alternatively) to new vertex v_i. That is every alternate edge of a path is replaced by C_3.

Theorem 2.4. *$A(T_n) \odot K_1$ is 3-difference cordial for all values of n .*

Proof. **Case 1.** Let the first triangle starts from u_1 and the last triangle ends with u_n. Here n is even. Let $V(A(T_n) \odot K_1) = V(A(T_n)) \cup \{x_i : 1 \leq i \leq n\} \cup \{w_i : 1 \leq i \leq \frac{n}{2}\}$ and $E(A(T_n) \odot K_1) = E(A(T_n)) \cup \{u_ix_i : 1 \leq i \leq n\} \cup \{v_iw_i : 1 \leq i \leq \frac{n}{2}\}$. In this case the order and size of $A(T_n) \odot K_1$ are 3n and $\frac{7n-2}{2}$. First we consider the path vertices u_i. Assign the labels 1,1,2,1 to the vertices u_1, u_2, u_3, u_4 respectively. Next we assign the labels 2,1,1,1 to the vertices u_5, u_6, u_7, u_8 respectively. Then we assign the labels 2,1,1,1 to the vertices $u_9, u_{10}, u_{11}, u_{12}$ respectively. Continuing this way assign the label to the next four vertices and so on. Clearly the last vertex u_n received the label 1. Consider the vertices v_i. Assign the label 1 to the vertices $v_1, v_3, v_5, v_7...$ Then we assign the label 2 to the vertices $v_2, v_4, v_6, v_8...$ Now we assign the label 3 to the vertices w_i $(1 \leq i \leq \frac{n}{2})$. Next we move to the vertices x_i. Assign the labels 2,2,3,3 to the vertices x_1, x_2, x_3, x_4 respectively. Then we assign the label 2 to the vertices $x_5, x_7, x_9, x_{11}....$ Finally assign the label 3 to the vertices $x_6, x_8, x_{10}, x_{12}...$ The vertex and edge condition of this case is $v_f(1) = v_f(2) = v_f(3) = n$ and

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 2 \pmod 4$	$\frac{7n-2}{4}$	$\frac{7n-2}{4}$
$n \equiv 0 \pmod 4$	$\frac{7n-4}{4}$	$\frac{7n}{4}$

Table 2.4:

Case 2. Let the first triangle starts from u_2 and the last triangle ends with u_{n-1}. Here also n is even. In this case the order and size of $A(T_n) \odot K_1$ are 3n-2 and $\frac{7n-8}{2}$. First we consider the path vertices u_i. Assign the labels 1,1,1 to the vertices u_1, u_2, u_3 respectively. Now we assign the labels 1,1,1,2 to the vertices u_4, u_5, u_6, u_7 respectively. Then we assign the labels 1,1,1,2 to the vertices u_8, u_9, u_{10}, u_{11} respectively. Proceeding like this assign the label to the next four vertices and so on. Note that in this case the last vertex u_n received the label 1. Next we move to the vertices v_i. Assign the label 2 to the vertex v_1. Then assign the label 2 to the vertices $v_2, v_4, v_6, v_8, ...$ and assign the label 1 to the vertices $v_3, v_5, v_7, v_9, ...$ Consider the vertices w_i. Assign the label to the vertices w_i $(1 \leq i \leq \frac{n}{2})$ as in case 1. Now we move to the vertices x_i. Assign the label 2 to the vertices x_1, x_2, x_5, x_7 and assign the label 3 to the vertices x_3, x_4, x_6. Assign the label 3 to all the vertices x_{4i} and x_{4i+1} for all the values of i=2,3,4,... For all the values of i=2,3,4,... assign the label to the vertices x_{4i+2} and x_{4i+3}. Clearly the last four vertices $x_{n-3}, x_{n-2}, x_{n-1}, x_n$ received the label by the integers 2,2,3,3. The vertex condition is $v_f(2) = v_f(3) = n-1$ and $v_f(1) = n$. Also the edge condition is $v_f(2) = v_f(3) = n-1$ and $v_f(1) = n$. Also the edge condition is given in table 2.5.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{7n-8}{4}$	$\frac{7n-8}{4}$
$n \equiv 2 \pmod 4$	$\frac{7n-10}{4}$	$\frac{7n-6}{4}$

Table 2.5:

Case 3. Let the first triangle starts from u_2 and the last triangle ends with u_n. Here n is odd. In this case the order and size of $A(T_n) \odot K_1$ are 3n-1 and $\frac{7n-5}{2}$ respectively. We consider the path vertices u_i. Assign the label 1 to the vertices u_1, u_2. Now we assign the labels 1,1,1,2 to the vertices u_3, u_4, u_5, u_6 respectively. Then we assign the labels 1,1,1,2 to the vertices u_7, u_8, u_9, u_{10} respectively. Continuing this way assign the label to the next four vertices and so on. Note that in this case the last vertex u_n received the label 1. Next we move to the vertices v_i. Assign the label to the vertices v_i $(1 \leq i \leq \lceil \frac{n}{2} \rceil)$ as in case 2. We consider the vertices w_i. Assign the lbel to the vertices w_i as in case 1. Next we move to the vertices x_i. Assign the label 3 to th vertices x_1 and x_2. Then we assign the label 2 to the vertex x_3. Assign the label 3 to the vertices x_{4i}, x_{4i+3} for all the values of i=1,2,3... For all the values of i=1,2,3,... assign the label 2 to the vertices x_{4i+1} and x_{4i+2}. Note that in this process the last four vertices $x_{n-3}, x_{n-2}, x_{n-1}, x_n$ received the label by the integers 3,2,2,3. The vertex condition is $v_f(2) = n - 1$ and $v_f(1) = v_f(3) = n$. Also the edge condition is given in table 2.6.

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 3 \pmod 4$	$\frac{7n-5}{4}$	$\frac{7n-5}{4}$
$n \equiv 1 \pmod 4$	$\frac{7n-7}{4}$	$\frac{7n-3}{4}$

Table 2.6:

□

Theorem 2.5. *$A(T_n) \odot 2K_1$ is 3-difference cordial.*

Proof. **Case 1.** Let the first triangle starts from u_1 and the last triangle ends with u_n. Here n is even. Let $V(A(T_n) \odot 2K_1) = V(A(T_n)) \cup \{x_i, x'_i : 1 \leq i \leq n\} \cup \{w_i, w'_i : 1 \leq i \leq \frac{n}{2}\}$ and $E(A(T_n) \odot 2K_1) = E(A(T_n)) \cup \{u_i x_i, u_i x'_i : 1 \leq i \leq n\} \cup \{v_i w_i, v_i w'_i : 1 \leq i \leq \frac{n}{2}\}$. In this case the order and size of $A(T_n) \odot 2K_1$ are $\frac{9n}{2}$ and 5n-1 respectively. Assign the labels 1,1 to the vertices u_1, u_2 respectively. Then we assign the path vertices u_3, u_5, u_7, u_9... labeled by the integer 1. Now we assign the label 2 to the vertices u_4, u_6, u_8, u_{10}... Consider the vertices v_i. Assign the label 2 to the vertex v_1. Then assign the label 1 to the vertices v_2, v_3, v_4, v_5... Now we move to the vertices w_i and w'_i. Assign the 1 to all the vertices of w_i $(1 \leq i \leq \frac{n}{2})$ and assign the label 3 to all the vertices of w'_i $(1 \leq i \leq \frac{n}{2})$. Finally we consider the vertices w_i and w'_i. Assign the label 2 to all the vertices of x_i $(1 \leq i \leq n)$. Next we assign the label 3 to all the vertices of x'_i $(1 \leq i \leq \frac{n}{2})$. The vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{3n}{2}$ and $e_f(0) = \frac{5n-2}{2}$ and $e_f(1) = \frac{5n}{2}$.

Case 2. Let the first triangle starts from u_2 and the last triangle ends with u_{n-1}. Here n is even. In this case the order and size of $A(T_n) \odot 2K_1$ are $\frac{9n-6}{2}$ and 5n-5 respectively. First we consider the path vertices u_i. Assign the label 1 to the vertices u_1, u_2, u_3 respectively. Then we assign the label 2 to the vertices $u_5, u_7, u_9, u_{11}, ...$ Next we assign the label 1 to the vertices $u_4, u_6, u_8, u_{10}...$ Note that in this case the last vertex u_n received the label 1. Then we move to the vertices v_i, w_i, w_i', x_i and x_i'. Assign the label to the vertices v_i $(1 \leq i \leq n-1)$, w_i $(1 \leq i \leq \frac{n}{2})$, w_i' $(1 \leq i \leq \frac{n}{2})$, x_i $(1 \leq i \leq n)$ and x_i' $(1 \leq i \leq n)$ as in case 1. The vertex and edge condition of this case is given by $v_f(1) = v_f(2) = v_f(3) = \frac{9n-6}{6}$ and $e_f(0) = \frac{5n-6}{2}$ and $e_f(1) = \frac{5n-4}{2}$.

Case 3. Let the first triangle starts from u_2 and the last triangle ends with u_n. Here n is odd. In this case the order and size of $A(T_n) \odot 2K_1$ are $\frac{9n-3}{2}$ and 5n-3 respectively. Consider the path vertices u_i. Assign the label 1 to the vertices $u_1, u_3, u_5, ...$ Then we assign the label 2 to the vertices $u_2, u_4, u_6, u_8, ...$ Clearly the last vertex u_n received the label 1. Next we consider the vertices v_i. Assign the label 1 to all the vertices of v_i $(1 \leq i \leq n-1)$. Next we move to the vertices w_i, w_i', x_i and x_i'. Assign the label to the vertices w_i,w_i' $(1 \leq i \leq \frac{n}{2})$ and x_i,x_i' $(1 \leq i \leq n)$ as in case 1. Since $v_f(1) = v_f(2) = v_f(3) = \frac{9n-3}{6}$ and $e_f(0) = e_f(1) = \frac{5n-3}{2}$, this labeling is a 3-difference cordial labeling. $\square$

Theorem 2.6. *$A(T_n) \odot K_2$ is 3-difference cordial.*

Proof. **Case 1.** Let the first triangle starts from u_1 and the last triangle ends with u_n. Here n is even. Let $V(A(T_n) \odot K_2) = V(A(T_n)) \cup \{x_i, x_i' : 1 \leq i \leq n\} \cup \{w_i, w_i' : 1 \leq i \leq \frac{n}{2}\}$ and $E(A(T_n) \odot K_2) = E(A(T_n)) \cup \{u_ix_i, u_ix_i', x_ix_i' : 1 \leq i \leq n\} \cup \{v_iw_i, v_iw_i', w_iw_i' : 1 \leq i \leq \frac{n}{2}\}$. In this case the order and size of $A(T_n) \odot K_2$ are $\frac{9n}{2}$ and $\frac{13n-2}{2}$ respectively. Consider the path vertices u_i. Assign the label 1 to the vertices u_1, u_2 respectively. Assign the labels 1,1,2,1 to the path vertices u_3, u_4, u_5, u_6 respectiely. Then we assign the labels 1,1,2,1 to the next four path vertices u_7, u_8, u_9, u_{10} respectively. Continuing like this we assign the label to the next four vertices and so on. Note that if all the vertices of u_i are labeled then we stop. Otherwise there are some non labeled vertices are exists. If th number of non labeled vertices are less than or equal to 3 then assign the labels 1,1,2 to the non labeled vertices. If it is two then we assign the labels 1,1 to the non lbeled vertices. If only one non labeled vertex is exist then assign the label 1 only. Next we assign the label 2 to all the vertices of v_i $(1 \leq i \leq n-1)$.

Consider the vertices w_i and w'_i. Assign the labels 2,3 to the vertices w_1, w'_1 respectivly. Assign the label 2 to the vertices w_2, w_4, w_6... and assign the label 1 to the vertices w_3, w_5, w_7... Then we asign the label 3 to all the vertices of w'_i $(1 \leq i \leq \frac{n}{2})$. Next we move to the vertices x_i and x'_i. Assign the label 2 to the vertices x_3, x_3, x_5... Then we assign the label 1 to the vertices x_2, x_4, x_6... Finally we asign the label 3 to all the vertices of x'_i $(1 \leq i \leq n)$. The vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{3n}{2}$. The edge condition is given in table 2.7.

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{13n}{4}$	$\frac{13n-4}{4}$
$n \equiv 2 \pmod 4$	$\frac{13n-2}{4}$	$\frac{13n-2}{4}$

Table 2.7:

Case 2. Let the first triangle starts from u_2 and the last triangle ends with u_{n-1}. Here n is even. In this case the order and size of $A(T_n) \odot K_2$ are $\frac{9n-6}{2}$ and $\frac{13n-12}{2}$ respectively. First we consider the path vertices u_i. Assign the label 1 to the vertex u_1. Now we assign the labels 1,1,1,2 to the path vertices u_2, u_3, u_4, u_5 respectiely. Then we assign the labels 1,1,1,2 to the next four path vertices u_6, u_7, u_8, u_9 respectively. Proceeding like this we assign the label to the next four vertices and so on. If all the vertices of u_i are labeled then we stop the process. Otherwise there are some non labeled vertices are exists. If th number of non labeled vertices are less than or equal to 3 then assign the labels 1,1,1 to the non labeled vertices. If it is two then we assign the labels 1,1 to the non labeled vertices. If only one non labeled vertex is exist then assign the label 1 only. Next we assign the label 2 to all the vertices of v_i. Assign the label to the vertices v_i $(1 \leq i \leq n-1)$ as in case 1. Now we move to the vertices w_i and w'_i. Assign the label 2 to the vertices w_1, w_3, w_5... and assign the label 1 to the vertices w_2, w_4, w_6... Then we assign the labels to the vertices w'_i $(1 \leq i \leq \frac{n}{2})$ as in case 1. Then we move to the vertices x_i and x'_i. Assign the label 2 to the vertices x_1 and x_2 respectively. Then we assign the label to the vertices x'_i $(1 \leq i \leq n)$ as in case 1. Next we assign the label 2 to the vertices x_{2i+1} for all the values of i=1,2,3,... For all the values of i=1,2,3,... assign the label 1 to the vertices x_{2i+2}. The vertex condition of this case is $v_f(1) = v_f(2) = v_f(3) = \frac{3n-2}{2}$. Also the edge condition of this case is given in table 2.8.

Nature of n	$e_f(0)$	$e_f(1)$
$n \equiv 0 \pmod 4$	$\frac{13n-12}{4}$	$\frac{13n-12}{4}$
$n \equiv 2 \pmod 4$	$\frac{13n-14}{4}$	$\frac{13n-10}{4}$

Table 2.8:

Case 3. Let the first triangle starts from u_2 and the last triangle ends with u_n. Here n is oddd. In this case the order and size of $A(T_n) \odot K_2$ are $\frac{9n-3}{2}$ and $\frac{13n-7}{2}$ respectively. First we consider the path vertices u_i. Assign the label 1 to the vertex u_1, u_2, u_3. Next we assign the labels 1,1,2,1 to the path vertices u_4, u_5, u_6, u_7 respectively. Then we assign the labels 1,1,2,1 to the next four path vertices u_8, u_9, u_{10}, u_{11} respectively. Continuing like this we assign the label to the next four vertices and so on. If all the vertices of u_i are labeled then we stop the process. Otherwise there are some non labeled vertices are exists. If th number of non labeled vertices are less than or equal to 3 then assign the labels 1,1,2 to the non labeled vertices. If it is two then we assign the labels 1,1 to the non labeled vertices. If only one non labeled vertex is exist then assign the label 1 only. Consider the vertices of v_i. Assign the label 2 to the vertex v_1. Then we assign the label 2 to the vertices v_2, v_4, v_6... and assign the label 1 to the vertices v_3, v_5, v_7... Now we move to the vertices w_i and w'_i. Assign the label 2 to all the vertices of w_i $(1 \leq i \leq \frac{n}{2})$ and assign the label 3 to all the vertices of w'_i $(1 \leq i \leq \frac{n}{2})$. Finally we consider the vertices x_i and x'_i. Assign the label 2 to the vertices x_1 and x_2. Next we assign the label to the vertices x'_i $(1 \leq i \leq n)$ as in case 1. Now we assign the label 1 to the vertices x_{2i+1} for all the values of i=1,2,3,... and assign the label 2 to the vertices x_{2i+2} for all the values of i=1,2,3,... The vertex condition is $v_f(1) = v_f(2) = v_f(3) = \frac{9n-3}{6}$. Also the edge condition of this case is given in table 2.9.

Values of n	$e_f(0)$	$e_f(1)$
$n \equiv 3 \pmod 4$	$\frac{13n-7}{4}$	$\frac{13n-7}{4}$
$n \equiv 1 \pmod 4$	$\frac{13n-5}{4}$	$\frac{13n-9}{4}$

Table 2.9:

Example 2.4. *A 3-difference cordial labeling of $A(T_4) \odot K_2$ with the first triangle starts from u_1 and the last triangle ends with u_n is given in figure 4].*

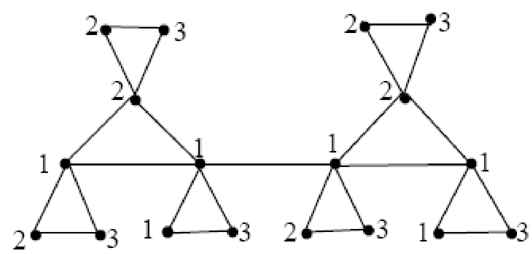

FIGURE 4

□

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