SEPARABILITY IN THE STRICT TOPOLOGY ON UNIFORM SPACE SETTING*

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Abstract

Let X be a Hausdorff uniform space, E a normed space, $C_{ub}(X, E)$ the space of all uniformly continuous, bounded and E – valued functions on X , $\mathcal{H}$ the class of all bounded and uniformly equicontinuous subset of $C_{ub}(X, E)$ and β_u the finest locally convex topology agreeing with the pointwise topology on each $H \in \mathcal{H}$. Among other things, it is studied the separability property of $(C_{ub}(X, E), \beta_u)$. It is proved that a such property is characterized in terms of X . As a consequence of that, it is shown that $(C_b(X, E), \beta_\infty)$, topology introduced by Khurana, has the separability property.

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1. Introduction

Let X be a Hausdorff uniform space, K the field of real or complex numbers, $C_{ub}(X)$ the vector space of all bounded K -valued uniformly continuous functions on X and $\mathcal{H}$ the class of all uniformly bounded and uniformly equicontinuous subsets of $C_{ub}(X)$. The uniform measures on X are defined to be linear K -valued functionals on $C_{ub}(X)$ which are continuous on each $H \in \mathcal{H}$ for the topology of pointwise convergence on X . This space is denoted by $M_u(X)$. Many authors ([2], [5], [12]) have studied these measures and have proved many measures-theoretic and topological properties. In $C_{ub}(X)$, it is defined β_u , the finest locally convex topology which agrees with the pointwise topology on each $H \in \mathcal{H}$, and is proved that the dual of $(C_{ub}(X), \beta_u)$ is $M_u(X)$. In this paper, we study the separability property of $(C_{ub}(X), \beta_u)$. We also extend this property to the vector-valued functions space.

2. Notations and Definitions

All linear spaces are over K . E will denote a normed space and $C_{ub}(X, E)$ the space of all uniformly continuous, bounded E -valued functions on X (if $E = K$, $C_{ub}(X, E)$ will be denoted by $C_{ub}(X)$). $\mathcal{H}$ will denote the class of all uniformly bounded, uniformly equicontinuous subsets of $C_{ub}(X, E)$ and any element of $\mathcal{H}$ will be identified by U.E.B.. For uniform spaces we refer to [9] and for locally convex spaces to [13]. The topology β_u on $C_{ub}(X, E)$ is defined to be the finest locally convex topology agreeing with pointwise topology on each $H \in \mathcal{H}$. For many others topologies defined on $C_{ub}(X, E)$ we use [14]. For an $f \in C_{ub}(X, E)$, $\|f\|$ is an element of $C_{ub}(X)$, where $\|f\|(x) = \|f(x)\|$.

The Banach Algebra $(C_{ub}(X), \|\cdot\|)$ gives, by Gelfan-Naimark theorem, a compactification of X , which we denote by $\overset{\vee}{X}$, and called the Samuel Compactification of X [6]. Each $f \in C_{ub}(X)$ has a unique extension to $\overset{\vee}{X}$ such that the canonical mapping $(C_{ub}(X), \|\cdot\|) \rightarrow (C_{ub}(\overset{\vee}{X}), \|\cdot\|)$ is an onto isometric isomorphism. Thus, a linear continuous mapping $\mu : (C_{ub}(X), \|\cdot\|) \rightarrow R$ gives rise to $\overset{\vee}{\mu} : (C_{ub}(X), \|\cdot\|) \rightarrow R$ which, by Riesz's representation theorem, can be considered as a regular Borel measure on $\overset{\vee}{X}$. If $\hat{X}$ is the completion of X , then one can assume that $X \subset \hat{X} \subset \overset{\vee}{X}$. If X is a complete metric space, then $\mu \in M_u(X)$ if and only if $\left| \overset{\vee}{\mu} \right|(\overset{\vee}{X} \setminus X) = 0$ (in [5]; it is proved for $K = R$, but easily extended to $K = C$), and so in this case $M_u(X) = M_t(X)$ (see [14]). For a general uniform space, let D be a family of uniformly continuous pseudo-metrics generating the unifor-

mity. For $d \in \mathcal{D}$, the quotient space with metric d is denoted by X_d ; its completion by $\hat{X}_d$, and the canonical mapping $X_d \rightarrow \hat{X}_d$ by φ_d . Let $\mu \in (C_{ub}(X), \|\cdot\|)'$. Then, $\mu \in M_u(X)$ if and only if $\varphi_d^* \mu \in M_u(\hat{X}) = M_t(\hat{X})$; here $\varphi_d^* \mu : C_{ub}(\hat{X}) \rightarrow K$ is defined by $\varphi_d^* \mu(f) = \mu(f \circ \varphi_d)$.

3. The Topology β_u

In this section, we shall give some results that will be needed in order to prove our main result. It is easy to see that, on $C_{ub}(X, E)$, β_u is coarser than $\|\cdot\|$ (the sup-norm).

Theorem 3.1. $C_{ub}(X) \otimes E$ is β_u -dense in $C_{ub}(X, E)$.

Proof. Take an absolutely convex β_u -neighborhood W of the origin. Fix $\epsilon > 0$, small enough such that

$$\{f \in C_{ub}(X, E) : \|f\| \leq \epsilon\} \subset \frac{1}{2}W.$$

Fix $f \in C_{ub}(X, E)$ with $\|f\| \leq \frac{1}{2}$. The pseudometric d on X , defined by $d(x, y) = \|f(x) - f(y)\|$, is uniformly continuous and $d(x, y) \leq 1$. For every $x \in X$, take $B(x, \epsilon) = \{y \in X : d(x, y) < \epsilon\}$ and define $g_x : X \rightarrow R$ by $g_x(y) = d(y, B(x, \epsilon))$. It is not hard to check that $g_x \in C_{ub}(X)$. Now, if I denotes the set of all finite subsets of X , then, for any $\alpha \in I$, $g_\alpha = \sup \{g_x : x \in \alpha\} \in C_{ub}(X)$. Also, $\{g_\alpha : \alpha \in I\}$ is a U.E.B subset of $C_{ub}(X)$, since $|g_\alpha(y) - g_\alpha(z)| \leq d(y, z)$. Moreover, $g_x(x) \geq \epsilon$ and $0 \leq g_\alpha(x) \leq 1$, for all $x \in X$. We define $g = \sup \{g_\alpha : \alpha \in I\}$. Then, $\epsilon \leq g \leq 1$, $\left\{\frac{g_\alpha}{g} : \alpha \in I\right\}$ is a U.E.B. set and $\frac{g_\alpha}{g} \rightarrow 1$ pointwise. From this, it follows that $\left\{\frac{g_\alpha}{g}f : \alpha \in I : \alpha \in I\right\}$ is a U.E.B subset of $C_{ub}(X, E)$ and $\frac{g_\alpha}{g}f \rightarrow f$ pointwise. Thus, there exists an $\alpha_o \in I$ such that $\frac{g_{\alpha_o}}{g}f - f \in \frac{1}{2}W$.

Assume $\alpha_o = \{x_1, x_2, \dots, x_n\}$ and define $h_i = g_{\{x_1, x_2, \dots, x_i\}} - g_{\{x_1, x_2, \dots, x_{i-1}\}}$, $2 \leq i \leq n$, $h_1 = g_{x_1}$. Thus, $g_{\alpha_o} = \sum_{i=1}^n h_i$.

Finally, we define $\sum_{i=1}^n \frac{h_i}{g} \otimes f(x_i)$, then, after some tedious calculus, we obtain that $\sum_{i=1}^n \frac{h_i}{g} \otimes f(x_i) - f \in W$ and the theorem.

In the follows, $\hat{X}$ will denote the uniform completion of the uniform space $(X, \mathcal{U})$. The next lemma will give us a powerful tool.

Lemma 3.1. *The canonical mapping $T : (C_{ub}(X, E), \beta_u) \rightarrow (C_{ub}(\hat{X}, E), \beta_u)$, $f \rightarrow \hat{f}$ is a continuous isomorphism.*

Proof. It follows easily from the facts that, for each $H \in \mathcal{H}$, $\hat{H} = \{\hat{f} : f \in H\}$ is a U.E.B subset of $C_{ub}(\hat{X}, E)$ and $T|_H, T|_{\hat{H}}^{-1}$ are both pointwise continuous.

The following result relates the topologies β_u and $\beta_{t|C_{ub}(X, E)}$, where β_t is the finest locally convex topology agreeing with the compact-open topology on each bounded subset of $C_b(X, E)$, (see [3])

Remark 3.1. .

- (i) $\beta_{t|C_{ub}(X, E)}$ is coarser than β_u . It follows easily from the fact that pointwise and compact-open topology coincide on each U .
- (ii) By the Stone-Weierstrass Theorem (see [14]; pp.155), $C_{ub}(X)$ is β_t -dense in $C_b(X)$.
- (iii) Choo ([3]) proved that $C_b(X) \otimes E$ is β_t -dense in $C_b(X, E)$

Theorem 3.2. $C_{ub}(X, E)$ is β_t -dense in $C_b(X, E)$

Proof. To prove this, it is enough, by remark 3.1 (iii), to prove that $C_{ub}(X) \otimes E$ is β_t -dense on $C_b(X) \otimes E$.

Note that $\Phi_e : C_b(X) \rightarrow C_b(X) \otimes E$, defined by $\Phi(f) = f \otimes e$, is a β_t -imbedding. Therefore, $\Phi(C_{ub}(X))$ is β_t -dense in $\Phi(C_b(X))$. Now, let's take

$$f = \sum_{i=1}^n \alpha_i f_i \otimes e_i \in C_b(X) \otimes E$$

and V a β_t -neighborhood of 0. Thus, there exist β_t -neighborhoods of 0 $W_1, \dots, W_n$ such that $W_1 + \dots + W_n \subset V$. For each $f_i \otimes e_i$ and for each W_i , we can choose $g_i \in C_{ub}(X)$ such that $f_i \otimes e_i - g_i \otimes e_i \in W_i$. From this, we have $\sum_{i=1}^n (f_i - g_i) \otimes e_i \in \sum_{i=1}^n W_i \subset V$ and the theorem.

4. The Uniform Measures

Now, we shall define the space of E -valued uniform measures. It is already known that if $(X, \mathcal{U})$ is a complete metric space, then, in the scalar case, $M_u(X) = M_t(X)$ ([2]).

Definition 4.1. Let $(X, \mathcal{U})$ be a uniform space. We define $M_u(X, E')$ as follows: $\mu \in M_u(X, E')$ if:

- (i) $\mu : C_{ub}(X) \rightarrow E'$ is linear
- (ii) The corresponding mapping $\mu : C_{ub}(X) \otimes E \rightarrow K$,

$$\mu\left(\sum_{i=1}^n f_i \otimes x_i\right) = \sum_{i=1}^n \langle \mu(f_i), x_i \rangle$$

is continuous for the $\|\cdot\|$ -topology on $C_{ub}(X) \otimes E$.

- (iii) $\mu_x : C_{ub}(X) \rightarrow K$, defined by $\mu_x(f) = \langle \mu(f), x \rangle$, is in $M_u(X)$ for every $x \in E$.

Remark 4.1. For $f_1, f_2 \in C_{ub}(X)^+$, and $\lambda \geq 0$, $|\mu|(f_1 + \lambda f_2) = |\mu|(f_1) + \lambda |\mu|(f_2)$, where $|\mu| : C_{ub}(X)^+ \rightarrow [0, \infty)$ is defined by

$$|\mu|(f) = \sup \{ |\mu(g)| : g \in C_{ub}(X) \otimes E; \|g\| \leq f \}.$$

The proof is a routine verification and is omitted.

Remark 4.2. As usual, we define $|\mu|(f) = |\mu|(f^+) - |\mu|(f^-)$, for any $f \in C_{ub}(X)$. It is easily verified that $|\mu|$ is linear bounded functional on $C_{ub}(X)$.

Theorem 4.1. If $\mu \in M_u(X, E')$, then $|\mu| \in M_u(X)$.

Proof. The mapping $\mu : C_{ub}(X) \otimes E \rightarrow K$ gives rise to the mapping $\check{\mu} : C(\check{X}) \otimes E \rightarrow K$ which can be continuously extended to $\check{\mu} : C(\check{X}, E) \rightarrow K$, with the uniform topology on $C(\check{X}, E)$ (Note $C(\check{X}) \otimes E$ is dense in $(C(\check{X}, E), \|\cdot\|)$).

Now, $\left| \check{\mu} \right|$ is regular Borel measure on $\check{X}$ and it is easy to see that $|\mu|^\vee = \left| \check{\mu} \right|$.

Let $\mathcal{U}$ be a uniform covering of X and $\mathcal{K}(\mathcal{U}) = \cup \left\{ \overset{\circ}{U} : U \in \mathcal{U} \right\}$, where closures and interiors are taken in $\overset{\vee}{X}$. By Frolik ([7], Th. 3, pp 405) it is enough to prove $|\mu|^\vee (\overset{\vee}{X} \setminus \mathcal{K}(\mathcal{U})) = 0$.

Take a Borel partition $\{F_i : 1 \leq i \leq n\}$ of $\overset{\vee}{X} \setminus \mathcal{K}(\mathcal{U})$ in $\overset{\vee}{X}$, and $\{\epsilon_i : 1 \leq i \leq n\}$ in the unit ball of E . Since $\mu_{\epsilon_i}(F_i) = 0$ for every i , we get $|\mu|^\vee (\overset{\vee}{X} \setminus \mathcal{K}(\mathcal{U})) = 0$. This proves the result.

In that follows we prove that $M_u(X, E')$ is the dual of $(C_{ub}(X, E), \beta_u)$. In order to see this, we first need to define the locally convex topology on $C_{ub}(X, E)$ generated by the seminorm $p_\mu(f) = |\mu|(\|f\|)$, for a fix $\mu \in M_u(X, E')$.

Lemma 4.1. *For $\mu \in M_u(X, E')$, $C_{ub}(X) \otimes E$ is dense in $(C_{ub}(X, E), p_\mu)$. Moreover, the unique continuous linear extension of μ to $(C_{ub}(X, E), p_\mu)$ satisfies*

$|\mu(f)| \leq |\mu|(\|f\|)$, for every $f \in C_{ub}(X, E)$ and is β_u -continuous.

Proof. Fix $\epsilon > 0$ and take $f \in C_{ub}(X, E)$. Proceeding as in Th. 3.1, we get a U.E.B. set $\{f_\alpha\}_{\alpha \in I}$ of $C_{ub}(X, E)$ such that $f_\alpha \rightarrow f$, pointwise on X and, for any fixed $\alpha_o \in I$, there exists $h \in C_{ub}(X) \otimes E$ such that $\|h - f_{\alpha_o}\| \leq \epsilon$. Since $|\mu| \in M_u(X)$, we get $|\mu|(\|f_\alpha - f\|) < \epsilon$, $\forall \alpha \geq \alpha_o$. Thus,

$$|\mu|(\|f - h\|) \leq |\mu|(\|f - f_{\alpha_o}\|) + |\mu|(\|f_{\alpha_o} - h\|) \leq \epsilon + \epsilon |\mu|(1)$$

and this proves denseness of $C_{ub}(X) \otimes E$ in $(C_{ub}(X, E), p_\mu)$.

Now, by the Hahn-Banach Theorem and the denseness of $C_{ub}(X) \otimes E$ in $(C_{ub}(X, E), p_\mu)$, μ has a unique extension and satisfies $|\mu(f)| \leq |\mu|(\|f\|)$ in $C_{ub}(X, E)$.

We will finally prove that μ is β_u -continuous. Take a U.E.B. net $\{f_\alpha\}_{\alpha \in I}$ with $f_\alpha \rightarrow 0$, in $C_{ub}(X, E)$. Using the facts that $\{\|f_\alpha\|\}$ is also U.E.B. net in $C_{ub}(X)$, $|\mu| \in M_u(X)$ and $|\mu(f_\alpha)| \leq |\mu|(\|f_\alpha\|)$, we get $\mu(f_\alpha) \rightarrow 0$ and the required continuity.

Theorem 4.2. *The dual of $(C_{ub}(X, E), \beta_u)$ is $M_u(X, E')$.*

Proof. Take $L \in (C_{ub}(X, E), \beta_u)'$ and put $\mu = L|_{C_{ub}(X) \otimes E}$. Fixing $\epsilon \in E$ and a U.E.B. net $\{f_\alpha\}$ in $C_{ub}(X)$, $f_\alpha \rightarrow 0$ pointwise, we get $f_\alpha \otimes \epsilon \rightarrow$

0 in β_u and so $\mu_\epsilon(f_\alpha) \rightarrow 0$, which proves $\mu \in M_u(X, E')$. Taking the p_μ - continuous linear extension μ , which is also β_u - continuous, we have $L = \mu$. The other side follows easily from previous Lemma.

5. Separability of $(C_{ub}(X), \beta_u)$

We start with the following definition.

Definition 5.1. A uniform space $(X, \mathcal{V})$ is said to be *separably uniformly submetrizable* if there exists a separable metric space M and a one - one uniformly continuous function h from X onto M .

Theorem 5.1. Let X be a separably uniformly submetrizable space. Then if $M_u(X) = M_t(X)$ (for example, if X is locally compact space or is a complete metrizable space), then $(C_{ub}(X), \beta_u)$ is separable.

Proof. If (M, d) is a separable metric space and h is a uniformly continuous and one-one function from X onto M , then we define $f_n(x) = d(h(x), y_n)$, where $\{y_n : n \in \mathbb{N}\}$ is a countable dense subset of M . It is clear that $f_n \in C_{ub}(X)$, for all $n \in \mathbb{N}$. If we define $f_0(x) = 1$ and if we denote by A the smallest sub algebra of $C_{ub}(X)$ which contains $\{f_n\}_{n=0}^\infty$, we have that this sub algebra A separates points and, for each $x \in X$, there exists $g \in A$ such that $g(x) \neq 0$. Therefore, by Th. 11 in [8], A is β_t - dense in $C_b(X)$. From this, it follows that $(C_{ub}(X), \beta_t)$ is separable.

On the other hand, since $M_u = (C_{ub}(X), \beta_u)' = (C_{ub}(X), \beta_t)' = M_t(X)$, the theorem follows from the fact that the separability property is possessed by all or none locally convex spaces with the same dual.

Corollary 5.1. If the uniformity is metrizable and X is separable, then $(C_{ub}(X), \beta_u)$ is separable.

Proof. If X is separable, then $\hat{X}$ is separable and, by the above theorem, $(C_{ub}(\hat{X}), \beta_u)$ is separable. Now, by Lemma 3.1, we have that $(C_{ub}(X), \beta_u)$ is separable.

Remark 5.1. If we consider the finest uniform structure $\mathcal{U}_f$ on X , then $C_{ub}(X) = C_b(X)$, $\mathcal{H}$ is the class of all pointwise equicontinuous and bounded sets of $C_b(X)$; therefore, the topology β_u is exactly the topology β_∞ defined

by Khurana [10]. Now, assuming the hypothesis of the above corollary, we get that $(C_b(X), \beta_\infty)$ has the separability property and so $(C_b(X), \beta_\tau)$ does it (the last result was proved by Wheeler [14]).

Theorem 5.2. *If $(C_{ub}(X), \beta_u)$ is separable, then $(X, \mathcal{U})$ is separably uniformly submetrizable.*

Proof. Let $\{g_n : n \in N\}$ be a β_u -dense subset of $C_{ub}(X)$. Let I_n be a compact interval such that $g_n(X) \subset I_n$, $n = 1, 2, \dots$. We define $e : X \rightarrow \prod_{n=1}^{\infty} I_n$, by $e(x) = (g_n(x))_{n=1}^{\infty}$. This function is one-one and uniformly continuous onto the separable metrics space $e(X)$.

6. Separability of $(C_{ub}(X, E), \beta_u)$

In the following Lemma we will use the ϵ -topology on the tensor product defined in ([13], p.96)

Lemma 6.1. *$C_{ub}(X) \otimes E$ is ϵ -dense in $C_b(X)_{\beta_t} \otimes_{\epsilon} E$, where $C_b(X)_{\beta_t}$ denotes the space $(C_b(X), \beta_t)$.*

Proof. This Lemma follows from the continuity of the function $\Phi : C_b(X)_{\beta_t} \times E \rightarrow C_b(X)_{\beta_t} \otimes_{\epsilon} E$, defined by $\Phi(f, e) = f \otimes e$, and the denseness of $C_{ub}(X)$ in $C_b(X)_{\beta_t}$.

In fact, if $(f_\alpha, e_\alpha) \rightarrow 0$ in the product topology of $C_b(X)_{\beta_t} \times E$, then $f_\alpha \xrightarrow{\beta_t} 0$ and $e_\alpha \rightarrow 0$. Since β_t is the topology of uniform convergence on β_t -equicontinuous subsets of $M_t(X)$, we have that $f_\alpha \rightarrow 0$ uniformly on each β_t -equicontinuous subset S of $M_t(X)$. Also, $e_\alpha \rightarrow 0$ on each $\|\cdot\|$ -equicontinuous subset T of E' .

Take $S \otimes T = \{\mu \otimes g : \mu \in S; g \in T\}$. Then

$$\begin{aligned} & \sup \{|\mu \otimes g(\Phi(f_\alpha, e_\alpha))| : \mu \otimes g \in S \otimes T\} \\ &= \sup \{|\mu \otimes g(f_\alpha \otimes e_\alpha)| : \mu \otimes g \in S \otimes T\} \\ &= \sup \{|\mu(f_\alpha)| |g(e_\alpha)| : \mu \otimes g \in S \otimes T\} \\ &\leq \sup \{|\mu(f_\alpha)| : \mu \in S\} \sup \{|g(e_\alpha)| : g \in T\} \rightarrow 0 \end{aligned}$$

Thus, $f_\alpha \otimes e_\alpha \rightarrow 0$ in the ϵ -topology of $C_b(X)_{\beta_t} \otimes_{\epsilon} E$.

Theorem 6.1. *If E is a separable Banach space, X is a separably uniformly submetrizable space and the uniformity is metrizable and complete on X , then $(C_{ub}(X, E), \beta_u)$ is separable.*

Proof. Since X is metrizable and complete uniform space, we have that $M_t(X, E') = M_u(X, E')$.

On the other hand, since $(C_b(X, E), \beta_t)' = M_t(X, E')$ (see [4]), $C'_{ub}(X, E')$ is β_t -dense in $C_b(X, E)$ (Th. 3.3) and $(C_{ub}(X, E), \beta_u)' = M_u(X, E')$, we have that $(C_{ub}(X, E), \beta_t)' = M_u(X, E')$. Therefore, to prove the statement it is enough to prove that $(C_{ub}(X, E), \beta_t)$ is separable.

By Th. 5.1, $(C_{ub}(X), \beta_t)$ is separable and by Lemma 6.1, the operator Φ is continuous. Thus, $\Phi(C_{ub}(X) \times E)$ is β_t -separable which is the linear hull of the β_t -dense subspace $C_{ub}(X) \otimes E$ of $C_{ub}(X, E)$. The theorem follows from this last fact.

Corollary 6.1. *If E is a separable Banach space, the uniformity on X is metrizable and X is separable, then $(C_{ub}(X, E), \beta_u)$ is separable.*

Theorem 6.2. *If $(C_{ub}(X, E), \beta_u)$ is separable, then X is separably uniformly submetrizable.*

Proof. Fix $g \in E'$, $g \neq 0$, and define $\Lambda : C_{ub}(X, E) \rightarrow C_{ub}(X)$ by $\Lambda(f) = g \circ f$. Λ is well defined, $\beta_u - \beta_u$ -continuous and onto. The statement follows from Th. 5.2.

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