

Proyecciones Journal of Mathematics
 Vol. 36, N° 4, pp. 601-614, December 2017.
 Universidad Católica del Norte
 Antofagasta - Chile

Some results on SD-Prime cordial labeling

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Received : February 2017. Accepted : March 2017

Abstract

Given a bijection $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$, we associate 2 integers $S = f(u) + f(v)$ and $D = |f(u) - f(v)|$ with every edge uv in $E(G)$. The labeling f induces an edge labeling $f' : E(G) \rightarrow \{0, 1\}$ such that for any edge uv in $E(G)$, $f'(uv) = 1$ if $\gcd(S, D) = 1$, and $f'(uv) = 0$ otherwise. Let $e_{f'}(i)$ be the number of edges labeled with $i \in \{0, 1\}$. We say f is SD-prime cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Moreover G is SD-prime cordial if it admits SD-prime cordial labeling. In this paper, we investigate the SD-prime cordial labeling of some derived graphs.

AMS Subject Classification 2010 : 05C78

Keywords : SD-prime labeling, SD-prime cordial labeling, star.

1. Introduction

Let $G = (V(G), E(G))$ be a simple, finite and undirected graph of order $|V(G)| = p$ and size $|E(G)| = q$. All notations not defined in this paper can be found in [2]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of vertices the labeling is called vertex labeling. If the domain is the set of edges, then we called about edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. For all detailed survey of graph labeling we refer Gallian [1]. In [4, 5], G.C. Lau and W.C. Shiu have introduced the concepts SD-prime labeling. In [3], G. C. Lau et.al. have introduced SD-prime cordial labeling and they proved behaviour of several graphs like path, complete bipartite graph, star, double star, wheel, fan, double fan and ladder are SD-prime cordial labeling. In this paper, we investigate the SD-prime cordial labeling behavior of $S'(K_{1,n})$, $D_2(K_{1,n})$, $S(K_{1,n})$, $DS(K_{1,n})$, $S'(B_{n,n})$, $D_2(B_{n,n})$, $DS(B_{n,n})$, $S(B_{n,n})$, $K_{1,3} * K_{1,n}$, CH_n , Fl_n , P_n^2 , $T(P_n)$, $T(C_n)$, the graph obtained by duplication of each vertex of path and cycle by an edge, Q_n , $A(T_n)$, TL_n , $P_n \odot K_1$, $C_n \odot K_1$ and J_n .

In [4], Lau and Shiu introduced a variant of prime graph labeling which is defined as follows.

Given a bijection $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$, we associate 2 integers $S = f(u) + f(v)$ and $D = |f(u) - f(v)|$ with every edge uv in E .

Definition 1.1. [4] A bijection $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ induces an edge labeling $f' : E(G) \rightarrow \{0, 1\}$ such that for any edge uv in G , $f'(uv) = 1$ if $\gcd(S, D) = 1$, and $f'(uv) = 0$ otherwise. We say f is SD-prime labeling if $f'(uv) = 1$ for all $uv \in E(G)$. Moreover, G is SD-prime if it admits SD-prime labeling.

Definition 1.2. [3] A bijection $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ induces an edge labeling $f' : E(G) \rightarrow \{0, 1\}$ such that for any edge uv in G , $f'(uv) = 1$ if $\gcd(S, D) = 1$, and $f'(uv) = 0$ otherwise. The labeling f is called SD-prime cordial labeling if $|e_{f'}(0) - e_{f'}(1)| \leq 1$. We say that G is SD-prime cordial if it admits SD-prime cordial labeling.

Definition 1.3. For every vertex $v \in V(G)$, the open neighbourhood set $N(v)$ is the set of all vertices adjacent to v in G .

Definition 1.4. For a graph G the splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that $N(v) = N(v')$.

Definition 1.5. The shadow graph $D_2(G)$ of a connected graph G is obtained by taking two copies of G , say G' and G'' . Join each vertex u' in G' to the neighbours of corresponding vertex u'' in G'' .

Definition 1.6. Duplication of a vertex v_k by a new edge $e = v'_k v''_k$ in a graph G produces a new graph G' such that $N(v'_k) \cap N(v''_k) = v_k$.

Definition 1.7. Let G be the a graph with $V = S_1 \cup S_2 \cup S_3 \cup \dots S_t \cup T$ where each S_i is a set of vertices having at least two vertices of the same degree and $T = V(G) \setminus \cup_{i=1}^t S_i$. The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2, \dots, w_t$ and joining to each vertex of S_i for $1 \leq i \leq t$.

Definition 1.8. For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G .

Definition 1.9. The subdivision graph $S(G)$ is obtained from G by subdividing each edge of G with a vertex.

Definition 1.10. The bistar $B_{n,n}$ is the graph obtained by attaching the apex vertices of two copies of $K_{1,n}$ by an edge.

Definition 1.11. $K_{1,3} * K_{1,n}$ is the graph obtained from $K_{1,3}$ by attaching root of a star $K_{1,n}$ at each pendant vertex of $K_{1,3}$.

Definition 1.12. The triangular ladder is a graph obtained from L_n by adding the edges $u_i v_{i+1}$, $1 \leq i \leq n-1$, where u_i and v_i , $1 \leq i \leq n$, are the vertices of L_n such that $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ are two paths of order n in the graph L_n .

Definition 1.13. The corona $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2 .

Definition 1.14. The total graph $T(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and two vertices are adjacent whenever they are either adjacent or incident in G .

2. Main results

Theorem 2.1. If G is SD-prime cordial of size q , then $G - e$ is also SD-prime cordial

- (i) for all $e \in E(G)$ when q is even.
- (ii) for some $e \in E(G)$ when q is odd.

Proof. **Case (i):** when q is even.

Let G be the SD-prime cordial graph of size q , where q is an even number. It follows that $e_{f'}(0) = e_{f'}(1) = \frac{q}{2}$. Let e be any edge in G which is labeled either 0 or 1. Then in $G - e$, we have either $e_{f'}(0) = e_{f'}(1) + 1$ or $e_{f'}(1) = e_{f'}(0) + 1$ and hence $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Thus $G - e$ is SD-prime cordial for all $e \in E(G)$.

Case (ii): when q is odd.

Let G be the SD-prime cordial graph of size q , where q is an odd number. It follows that either $e_{f'}(0) = e_{f'}(1) + 1$ or $e_{f'}(1) = e_{f'}(0) + 1$. If $e_{f'}(0) = e_{f'}(1) + 1$ then remove an edge e which is labeled as 0 and if $e_{f'}(1) = e_{f'}(0) + 1$ then remove an edge e which is labeled as 1 from G . It follows that $e_{f'}(0) = e_{f'}(1)$. Thus, $G - e$ is SD-prime cordial for some $e \in E(G)$.
□

Corollary 2.2. The graph $G + e$ is SD-prime cordial if G is SD-prime cordial having even size.

Theorem 2.3. The graph $S'(K_{1,n})$ is SD-prime cordial.

Proof. Let $v_1, v_2, \dots, v_n$ be the pendant vertices and v be the apex vertex of $K_{1,n}$ and $u, u_1, u_2, \dots, u_n$ are added vertices corresponding to $v, v_1, v_2, \dots, v_n$ to obtain $S'(K_{1,n})$. Therefore, $S'(K_{1,n})$ is of order $2n + 2$ and size $3n$. Define $f : V(S'(K_{1,n})) \rightarrow \{1, 2, \dots, 2n + 2\}$ as follows:

$$\begin{aligned}
 f(v) &= 4; \\
 f(u) &= 1; \\
 f(v_i) &= \begin{cases} 2i+1 & \text{if } i \text{ is odd and } 1 \leq i \leq n \\ 2i+2 & \text{if } i \text{ is even and } 1 \leq i \leq n; \end{cases} \\
 f(u_i) &= \begin{cases} 2 & \text{if } i = 1 \\ 2i+2 & \text{if } i \text{ is odd and } 2 \leq i \leq n \\ 2i+1 & \text{if } i \text{ is even and } 2 \leq i \leq n; \end{cases}
 \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(1) = \left\lfloor \frac{3n}{2} \right\rfloor$ and $e_{f'}(0) = \left\lceil \frac{3n}{2} \right\rceil$.

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $S'(K_{1,n})$ is SD-prime cordial. $\square$

Theorem 2.4. *The graph $D_2(K_{1,n})$ is SD-prime cordial.*

Proof. Let $u, u_1, u_2, \dots, u_n$ and $v, v_1, v_2, \dots, v_n$ be the vertices of two copies of $K_{1,n}$. Let $V(D_2(K_{1,n})) = \{u, v\} \cup \{u_i, v_i : 1 \leq i \leq n\}$ and $E(D_2(K_{1,n})) = \{uu_i, vv_i, uv_i, vu_i : 1 \leq i \leq n\}$. Therefore, $D_2(K_{1,n})$ is of order $2n+2$ and size $4n$. Define $f : V(D_2(K_{1,n})) \rightarrow \{1, 2, \dots, 2n+2\}$ as follows:

$$\begin{aligned}
 f(u) &= 1; \\
 f(v) &= 2; \\
 f(u_i) &= \begin{cases} 2i+1 & \text{if } i \text{ is odd and } 1 \leq i \leq n \\ 2i+2 & \text{if } i \text{ is even and } 1 \leq i \leq n; \end{cases} \\
 f(v_i) &= \begin{cases} 2i+2 & \text{if } i \text{ is odd and } 1 \leq i \leq n \\ 2i+1 & \text{if } i \text{ is even and } 1 \leq i \leq n. \end{cases}
 \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = e_{f'}(1) = 2n$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $D_2(K_{1,n})$ is SD-prime cordial. $\square$

Theorem 2.5. *The graph $S(K_{1,n})$ is SD-prime cordial.*

Proof. Let $v, v_1, v_2, \dots, v_n$ be the vertices of $K_{1,n}$. Let $V(S(K_{1,n})) = \{v\} \cup \{v_i, u_i : 1 \leq i \leq n\}$ and $E(S(K_{1,n})) = \{vu_i, v_i u_i : 1 \leq i \leq n\}$. Therefore, $S(K_{1,n})$ is of order $2n+1$ and size $2n$. Define $f : V(S(K_{1,n})) \rightarrow \{1, 2, \dots, 2n+1\}$ as follows:

$$\begin{aligned}
 f(v) &= 1; \\
 f(u_i) &= 2i+1, \quad 1 \leq i \leq n; \\
 f(v_i) &= 2i, \quad 1 \leq i \leq n.
 \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = e_{f'}(1) = n$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence $S(K_{1,n})$ is SD-prime cordial. $\square$

Theorem 2.6. *The graph $S'(B_{n,n})$ is SD-prime cordial.*

Proof. Let $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ be the pendant vertices and u, v be the apex vertices of $B_{n,n}$. Let u', v', u'_i, v'_i be added vertices corresponding to u, v, u_i, v_i to obtain $S'(B_{n,n})$. Therefore, $S'(B_{n,n})$ is of order $4n + 4$ and size $6n + 3$. Define $f : V(S'(B_{n,n})) \rightarrow \{1, 2, \dots, 4n + 4\}$ as follows:

$$\begin{aligned} f(u) &= 6; \\ f(v) &= 2; \\ f(u') &= 3; \\ f(v') &= 1; \\ f(u_i) &= \begin{cases} 4 & \text{if } i = 1 \\ 4i + 4 & \text{if } i \equiv 1 \pmod{3} \text{ and } 2 \leq i \leq n \\ 4i + 2 & \text{if } i \equiv 0, 2 \pmod{3} \text{ and } 2 \leq i \leq n; \end{cases} \\ f(u'_i) &= \begin{cases} 8 & \text{if } i = 1 \\ 4i + 2 & \text{if } i \equiv 1 \pmod{3} \text{ and } 2 \leq i \leq n \\ 4i + 4 & \text{if } i \equiv 0, 2 \pmod{3} \text{ and } 2 \leq i \leq n; \end{cases} \\ f(v_i) &= 5 + 4(i - 1), \quad 1 \leq i \leq n; \\ f(v'_i) &= 7 + 4(i - 1), \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = \left\lfloor \frac{6n+3}{2} \right\rfloor$ and $e_{f'}(1) = \left\lceil \frac{6n+3}{2} \right\rceil$.

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $S'(B_{n,n})$ is SD-prime cordial. $\square$

Theorem 2.7. *The graph $D_2(B_{n,n})$ is SD-prime cordial.*

Proof. Let $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ be the pendant vertices and u, v be the apex vertices of $B_{n,n}$. Let

$$\begin{aligned} V(D_2(B_{n,n})) &= \{u, v, u', v'\} \cup \{u_i, v_i, u'_i, v'_i : 1 \leq i \leq n\} \text{ and } E(D_2(B_{n,n})) \\ &= \{uv, u'v, uv', u'v'\} \cup \{uu_i, uu'_i, u'u'_i, u'u_i, vv_i, vv'_i, v'v_i, v'v'_i : 1 \leq i \leq n\}. \end{aligned}$$

Therefore, $D_2(B_{n,n})$ is of order $4n + 4$ and size $8n + 4$. Define $f : V(D_2(B_{n,n})) \rightarrow \{1, 2, \dots, 4n + 4\}$ as follows:

Let p be the highest prime number $< 4n + 4$.

$$\begin{aligned} f(u) &= 1; \\ f(v) &= p; \\ f(u') &= 2; \\ f(v') &= 4; \\ f(u'_i) &= 4i + 2, \quad 1 \leq i \leq n; \\ f(v_i) &= 4i + 4, \quad 1 \leq i \leq n. \end{aligned}$$

For the vertices $u_1, u_2, \dots, u_n, v'_1, v'_2, \dots, v'_n$ we assign distinct odd numbers except 1 and p . In view of the above labeling pattern we get, $e_{f'}(0) = e_{f'}(1) = 4n + 2$.

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $D_2(B_{n,n})$ is SD-prime cordial. $\square$

Theorem 2.8. *The graph $DS(B_{n,n})$ is SD-prime cordial.*

Proof. Let $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ be the pendant vertices and u, v be the apex vertices of $B_{n,n}$. Let $V(DS(B_{n,n})) = \{u, v, w_1, w_2\} \cup \{u_i, v_i : 1 \leq i \leq n\}$ and $E(DS(B_{n,n})) = \{uv, uw_2, vw_2\} \cup \{uu_i, vv_i, u_iw_1, v_iw_1 : 1 \leq i \leq n\}$. Therefore, $DS(B_{n,n})$ is of order $2n + 4$ and size $4n + 3$. Define $f : V(DS(B_{n,n})) \rightarrow \{1, 2, \dots, 2n + 4\}$ as follows:

$$\begin{aligned} f(u) &= 4; \\ f(v) &= 2; \\ f(w_1) &= 1; \\ f(w_2) &= 2n + 3; \\ f(u_i) &= 6 + 2(i - 1), \quad 1 \leq i \leq n; \\ f(v_i) &= 3 + 2(i - 1), \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = 2n + 1$ and $e_{f'}(1) = 2n + 2$.

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $DS(B_{n,n})$ is SD-prime cordial. $\square$

Theorem 2.9. *The graph $S(B_{n,n})$ is SD-prime cordial.*

Proof. Let $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ be the pendant vertices and u, v be the apex vertices of $B_{n,n}$. Let v'_i and u'_i be the newly added vertices between v and v_i and u and u_i respectively. Also, w be the newly added vertex between u and v . Therefore, $S(B_{n,n})$ is of order $4n + 3$ and size

$4n + 2$. Define $f : V(S(B_{n,n})) \rightarrow \{1, 2, \dots, 4n + 3\}$ as follows:

$$\begin{aligned} f(u) &= 1; \\ f(v) &= 2; \\ f(w) &= 3; \\ f(u_i) &= 4i, \quad 1 \leq i \leq n; \\ f(u'_i) &= 4i + 1, \quad 1 \leq i \leq n; \\ f(v_i) &= 4i + 3, \quad 1 \leq i \leq n; \\ f(v'_i) &= 4i + 2, \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = e_{f'}(1) = 2n + 1$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $S(B_{n,n})$ is SD-prime cordial. $\square$

Theorem 2.10. *The graph $K_{1,3} * K_{1,n}$ is SD-prime cordial.*

Proof. Let $V(K_{1,3} * K_{1,n}) = \{x, u, v, w\} \cup \{u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E(K_{1,3} * K_{1,n}) = \{xu, xv, xw\} \cup \{uu_i, vv_i, ww_i : 1 \leq i \leq n\}$. Therefore, $K_{1,3} * K_{1,n}$ is of order $3n + 4$ and size $3n + 3$. Define $f : V(K_{1,3} * K_{1,n}) \rightarrow \{1, 2, \dots, 3n + 4\}$ as follows:

$$\begin{aligned} f(u) &= 1; \\ f(v) &= 2; \\ f(w) &= 4; \\ f(x) &= 3; \\ f(u_i) &= 3i + 2, \quad 1 \leq i \leq n; \\ f(v_i) &= 3i + 3, \quad 1 \leq i \leq n; \\ f(w_i) &= 3i + 4, \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get,

$$\begin{aligned} e_{f'}(0) &= \begin{cases} \frac{3n+3}{2} & \text{if } n \text{ is odd} \\ \frac{3n+2}{2} & \text{if } n \text{ is even} \end{cases} \\ e_{f'}(1) &= \begin{cases} \frac{3n+3}{2} & \text{if } n \text{ is odd} \\ \frac{3n+4}{2} & \text{if } n \text{ is even} \end{cases} \end{aligned}$$

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $K_{1,3} * K_{1,n}$ is SD-prime cordial. $\square$

Theorem 2.11. *The triangular ladder TL_n is SD-prime cordial.*

Proof. Let $v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ be the vertices of L_n . Let $V(TL_n) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(TL_n) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$. Therefore, TL_n is of order $2n$ and size $4n-3$. Define $f : V(TL_n) \rightarrow \{1, 2, \dots, 2n\}$ as follows:

$$\begin{aligned} f(u_i) &= 2i, \quad 1 \leq i \leq n; \\ f(v_i) &= 2i-1, \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = 2n-2$ and $e_{f'}(1) = 2n-1$.

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, TL_n is SD-prime cordial. $\square$

Theorem 2.12. *The closed helm graph CH_n is SD-prime cordial.*

Proof. Let v be an apex vertex and $v_1, v_2, \dots, v_n$ are rim vertices of W_n . Let $u_1, u_2, \dots, u_n$ be the pendant vertices which are joined to each rim vertices of W_n to obtained H_n . So $V(CH_n) = \{v\} \cup \{v_i, u_i : 1 \leq i \leq n\}$ and $E(CH_n) = \{vv_i, u_i v_i : 1 \leq i \leq n\} \cup \{v_i v_{i+1}, u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{v_1 v_n, u_1 u_n\}$. Therefore, CH_n is of order $2n+1$ and size $4n$. Define $f : V(CH_n) \rightarrow \{1, 2, \dots, 2n+1\}$ as follows:

$$\begin{aligned} f(v) &= 1; \\ f(v_i) &= 2i, \quad 1 \leq i \leq n; \\ f(u_i) &= 2i+1, \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = e_{f'}(1) = 2n$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, CH_n is SD-prime cordial. $\square$

Theorem 2.13. *The flower Fl_n is SD-prime cordial.*

Proof. Let $V(Fl_n) = \{v\} \cup \{v_i, u_i : 1 \leq i \leq n\}$ and $E(Fl_n) = \{vv_i, v_i u_i, v u_i : 1 \leq i \leq n\} \cup \{v_n v_1\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\}$. Therefore, Fl_n is of order $2n+1$ and size $4n$. Define $f : V(Fl_n) \rightarrow \{1, 2, \dots, 2n+1\}$ as follows. Assign the labels to vertices v, v_i, u_i as in theorem 2.14. We observe that, $e_{f'}(0) = e_{f'}(1) = 2n$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, Fl_n is SD-prime cordial. $\square$

Theorem 2.14. *The graph P_n^2 is SD-prime cordial.*

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of the path P_n . Let $V(P_n^2) = \{v_1, v_2, \dots, v_n\}$ and $E(P_n^2) = \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i v_{i+2} : 1 \leq i \leq n-2\}$. Therefore, P_n^2 is of order n and size $2n-3$. We define $f : V(P_n^2) \rightarrow \{1, 2, \dots, n\}$ by $f(v_i) = i$ for $1 \leq i \leq n$. We observe that, $e_{f'}(0) = n-2$ and $e_{f'}(1) = n-1$.

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, P_n^2 is SD-prime cordial. $\square$

Remark 2.15. Note that $T(P_n) = P_{2n-1}^2$. Therefore, $T(P_n)$ is also SD-prime cordial.

Theorem 2.16. The graph obtained by duplication of each vertex by an edge in P_n is SD-prime cordial.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of the path P_n and G be the graph obtained by duplication of each vertex v_i of the path P_n by an edge $v_i' v_i''$ for $1 \leq i \leq n$ at a time. Let $V(G) = \{v_i, v_i', v_i'' : 1 \leq i \leq n\}$ and $E(G) = \{v_i v_i', v_i v_i'', v_i' v_i'' : 1 \leq i \leq n\} \cup \{v_i v_{i+1} : 1 \leq i \leq n-1\}$. Therefore, G is of order $3n$ and size $4n-1$. Define $f : V(G) \rightarrow \{1, 2, \dots, 3n\}$ as follows:

$$\begin{aligned} f(v_{2i-1}) &= 6i-4, \quad 1 \leq i \leq \lceil \frac{n}{2} \rceil; \\ f(v_{2i}) &= 6i-2, \quad 1 \leq i \leq \lfloor \frac{n}{2} \rfloor; \\ f(v_{2i-1}') &= 6i-5, \quad 1 \leq i \leq \lceil \frac{n}{2} \rceil; \\ f(v_{2i}') &= 6i-1, \quad 1 \leq i \leq \lfloor \frac{n}{2} \rfloor; \\ f(v_i'') &= 3i, \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(1) = 2n$ and $e_{f'}(0) = 2n-1$.

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, G is SD-prime cordial. $\square$

Theorem 2.17. The graph $T(C_n)$ is SD-prime cordial.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of the cycle C_n . Let $V(T(C_n)) = \{v_i, u_i : 1 \leq i \leq n\}$ and $E(T(C_n)) = \{v_i v_{i+1}, u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i u_i : 1 \leq i \leq n\} \cup \{v_i u_{i-1} : 2 \leq i \leq n\} \cup \{v_n v_1, u_n u_1, v_1 u_n\}$. Therefore, $T(C_n)$ is of order $2n$ and size $4n$. Define $f : V(T(C_n)) \rightarrow \{1, 2, \dots, 2n\}$ as follows:

$$\begin{aligned} f(v_i) &= 2i-1; \quad 1 \leq i \leq n \\ f(u_i) &= 2i; \quad 1 \leq i \leq n \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = e_{f'}(1) = 2n$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $T(C_n)$ is SD-prime cordial. $\square$

Remark 2.18. Note that $T(C_n) = C_{2n}^2$. It is natural to determine the SD-prime cordiality of C_{2n+1}^2 , $n \geq 2$.

Theorem 2.19. The graph obtained by duplication of each vertex by an edge in C_n is SD-prime cordial.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of the cycle C_n and G be the graph obtained by duplication of each vertex v_i of the cycle C_n by an edge $v_i'v_i''$ for $1 \leq i \leq n$. Then $V(G) = \{v_i, v_i', v_i'' : 1 \leq i \leq n\}$ and $E(G) = E(C_n) \cup \{v_i v_i', v_i v_i'', v_i' v_i'' : 1 \leq i \leq n\}$. Therefore, G is of order $3n$ and size $4n$. Define $f : V(G) \rightarrow \{1, 2, \dots, 3n\}$ as follows:

$$\begin{aligned} f(v_{2i-1}) &= 6i - 5, \quad 1 \leq i \leq \lceil \frac{n}{2} \rceil; \\ f(v_{2i}) &= 6i - 1, \quad 1 \leq i \leq \lfloor \frac{n}{2} \rfloor; \\ f(v_{2i-1}') &= 6i - 4, \quad 1 \leq i \leq \lceil \frac{n}{2} \rceil; \\ f(v_{2i}'') &= 6i - 2, \quad 1 \leq i \leq \lfloor \frac{n}{2} \rfloor; \\ f(v_i'') &= 3i, \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = e_{f'}(1) = 2n$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, G is SD-prime cordial. $\square$

Theorem 2.20. The quadrilateral snake Q_n is SD-prime cordial.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of path P_n . Let $V(Q_n) = V(P_n) \cup \{u_i, w_i : 1 \leq i \leq n-1\}$ and $E(Q_n) = E(P_n) \cup \{v_i u_i, u_i w_i, v_{i+1} w_i : 1 \leq i \leq n-1\}$. Therefore, Q_n is of order $3n-2$ and size $4n-4$. Define $f : V(Q_n) \rightarrow \{1, 2, \dots, 3n-2\}$ as follows:

$$\begin{aligned} f(v_i) &= \begin{cases} 3i-2 & \text{if } i \text{ is odd and } 1 \leq i \leq n \\ 3i-3 & \text{if } i \text{ is even and } 1 \leq i \leq n \end{cases}; \\ f(u_i) &= 3i-1, \quad 1 \leq i \leq n-1; \\ f(w_i) &= \begin{cases} 3i+1 & \text{if } i \text{ is odd and } 1 \leq i \leq n-1 \\ 3i & \text{if } i \text{ is even and } 1 \leq i \leq n-1. \end{cases} \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = e_{f'}(1) = 2n-2$. Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, Q_n is SD-prime cordial. $\square$

Theorem 2.21. An alternate triangular snake $A(T_n)$ is SD-prime cordial.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of path P_n . The graph $A(T_n)$ is obtained by joining the vertices $v_i v_{i+1}$ (alternately) to a new vertex $u_i, 1 \leq i \leq n-1$ for even n and $1 \leq i \leq n-2$ for odd n . Therefore, $V(A(T_n)) = V(P_n) \cup \{u_i : 1 \leq i \leq \lfloor \frac{n}{2} \rfloor\}$ and $E(A(T_n)) = E(P_n) \cup \{v_{2i-1} u_i : 1 \leq i \leq \lfloor \frac{n}{2} \rfloor; u_i v_{2i} : 1 \leq i \leq \lfloor \frac{n}{2} \rfloor\}$. Also,

$$|V(A(T_n))| = \begin{cases} \frac{3n-1}{2} & \text{if } n \text{ is odd} \\ \frac{3n}{2} & \text{if } n \text{ is even} \end{cases}$$

$$|E(A(T_n))| = \begin{cases} 2n-2 & \text{if } n \text{ is odd} \\ 2n-1 & \text{if } n \text{ is even} \end{cases}$$

Case 1: n is odd.

Define $f : V(A(T_n)) \rightarrow \{1, 2, \dots, \frac{3n-1}{2}\}$ as follows:

$$f(v_i) = \begin{cases} 3i-2 & \text{if } i \text{ is odd} \\ 3i-1 & \text{if } i \text{ is even} ; \end{cases}$$

$$f(u_i) = 3i, 1 \leq i \leq \lfloor \frac{n}{2} \rfloor.$$

We observe that, $e_{f'}(0) = e_{f'}(1) = n-1$.

Case 2: n is even.

Define $f : V(A(T_n)) \rightarrow \{1, 2, \dots, \frac{3n}{2}\}$ as follows:

$$f(v_i) = \begin{cases} 3i-1 & \text{if } i \text{ is odd} \\ 3i & \text{if } i \text{ is even} ; \end{cases}$$

$$f(u_i) = 3i-2, 1 \leq i \leq \lfloor \frac{n}{2} \rfloor.$$

We observe that, $e_{f'}(0) = n-1$ and $e_{f'}(1) = n$.

Thus, in both cases $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence, $A(T_n)$ is SD-prime cordial.

□

Theorem 2.22. The comb $P_n \odot K_1$ is SD-prime cordial.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of path P_n . Let $V(P_n \odot K_1) = \{v_i, u_i : 1 \leq i \leq n\}$ and $E(P_n \odot K_1) = \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i u_i : 1 \leq i \leq n\}$. Therefore, $P_n \odot K_1$ is of order $2n$ and size $2n-1$. Define

$f : V(P_n \odot K_1) \rightarrow \{1, 2, \dots, 2n\}$ as follows:

$$\begin{aligned} f(v_i) &= 2i - 1, \quad 1 \leq i \leq n; \\ f(u_i) &= 2i, \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = n - 1$ and $e_{f'}(1) = n$.

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence $P_n \odot K_1$ is SD-prime cordial. $\square$

Corollary 2.23. *The crown $C_n \odot K_1$ is SD-prime cordial.*

Theorem 2.24. *The jewel J_n is SD-prime cordial.*

Proof. Let $V(J_n) = \{u, v, x, y\} \cup \{u_i : 1 \leq i \leq n\}$ and $E(J_n) = \{ux, uy, xy, vx, vy\} \cup \{uu_i, vu_i : 1 \leq i \leq n\}$. Therefore, J_n is of order $n + 4$ and size $2n + 5$. Define $f : V(J_n) \rightarrow \{1, 2, \dots, n + 4\}$ as follows:

$$\begin{aligned} f(u) &= 1; \\ f(v) &= 2; \\ f(x) &= 3; \\ f(y) &= 4; \\ f(u_i) &= i + 4, \quad 1 \leq i \leq n. \end{aligned}$$

In view of the above labeling pattern we get, $e_{f'}(0) = n + 2$ and $e_{f'}(1) = n + 3$.

Thus, $|e_{f'}(0) - e_{f'}(1)| \leq 1$. Hence J_n is SD-prime cordial. $\square$

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