Proyecciones Journal of Mathematics
Vol. 29, N^o 3, pp. 181-191, December 2010.
Universidad Católica del Norte
Antofagasta - Chile
DOI: 10.4067/S0716-09172010000300002

A NEW DEFINITION OF S^* CLOSEDNESS IN $L-$TOPOLOGICAL SPACES

BO CHEN
SOUTHWEST UNIVERSITY, P. R. CHINA
Received : June 2009. Accepted : May 2010

Abstract

In this paper, a new notion of S^* closedness in L-topological Spaces is introduced by means of semi-open $L-$sets and their inequality where L is a complete $DeMorgan$ algebra.This new definition doesn't rely on the structure of basic lattice L. It can be characterized by means of semi-open $L-$sets and their inequality . When L is completely distributive $DeMorgan$ algebra, its many characterizations are presented.

Keywords and Phrases : *semiopen $L-set$, S^* closedness, $L-$topological space.*

Mathematics Subject Classification(2000) : *54A40,54A20.*

1. Introduction

Compactness is one of the most important notions in topology. In [1], Azad introduced the concepts of semiopen sets and semiopen continuous functions in $L-$ topology. In [5], Kudri generalized the concept of semicompactness and S^* closedness in $L-$ topological spaces. However, Kudri's S^* closedness relies on the structure of L which is a completely distributive lattice.

In [10], Shi introduced a new definition of fuzzy compactness in $L-$ topological spaces by means of open $L-$ sets and their inequality where L is a completely distributive $DeMorgan$ algebra. This new definition doesn't rely on the structure of basic lattice L.

In this paper, along the line of [10] we shall introduce a new definition of S^* closedness in $L-$ topological spaces by means of semiopen $L-$ sets and their inequality. Also we give characterizations and properties of S^* closedness .

2. Preliminaries

Throughout this paper, $(L, \bigvee, \bigwedge, ')$ is a complete $DeMorgan$ algebra, X is a nonempty set. L^X is the set of all $L-$ fuzzy sets ($L-$sets for short) on X. The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$.

An element a in L is called prime element if $a \geq b \bigwedge c$ implies $a \geq b$ or $a \geq c$. a in L is called co-prime element if a' is a prime element. The set of non-unit prime elements in L is denoted by $P(L)$. The set of non-zero co-prime elements in L is denoted by $M(L)$.

The binary relation $\prec$ in L is defined as follows: for $a, b \in L, a \prec b$ if and only if for every subset $D \subset L$, the relation $b \leq \sup D$ always implies the existence of $d \in D$ with $a \leq d$. In a complete distributive $DeMorgan$ algebra L, each element b is a sup of $\{a \in L | a \prec b\}$, $\{a \in L | a \prec b\}$is called the greatest minimal family of b in the sense of [16], in symbol $\beta(b)$. Moreover for $b \in L$, define $\beta^*(b) = \beta(b) \bigcap M(L), \alpha(b) = \{a \in L | a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \bigcap P(L)$.

For $a \in L$ and $A \in L^X$, let

$$A_{[a]} = \{x \in X | A(x) \geq a\},$$

$$A_{(a)} = \{x \in X | a \in \beta(A(x))\},$$

$$A^{(a)} = \{x \in X | A(x) \not\leq a\}.$$

An $L-$topological space ($L-$ space for short) is a pair (X, δ), where δ is a subfamily of L^X which contains $\underline{0}$ $\underline{1}$ and is closed for any suprema and infima. δ is called an $L-$topology on X. Every member of δ is called an open $L-$set and its quasicomplementation is called a closed $L-$set.

If (X, τ) is a topological space, then $\omega(\tau)$ denote the $L-$topology of the continuous functions from (X, τ) to L with its Scott topology. $(X, \omega(\tau))$ is called to be topologically generated by (X, δ).

For a subfamily $\Phi \subseteq L^X, 2^{(\Phi)}$ denotes the set of all finite subfamily of Φ.

Definition 2.1 ([1, 9]) Let (X, δ) be an $L-$space. $A \in L^X$ is called semiopen if and only if there exists $B \in \delta$ such that $B \leq A \leq cl(B)$. $A \in L^X$ is called semiclosed if and only if $A^{'}$ is semiopen.

Definition 2.2([1, 9]) Let (X, δ) be an $L-$space, $A \in L^X$. The set $cl_*A = \bigwedge\{B \in L^X | A \leq B$ and B is semiclosed $\}$is called the semiclosure of A. $int_*A = \bigvee\{B \in L^X | B \leq A$ and B is semiopen $\}$is called the semiinterior of A.

Definition 2.3([10]) Let (X, δ) be an $L-$space. $G \in L^X$ is called compact if for every family $\mathcal{U}$ of open $L-$ sets, it follows that

$$\bigwedge_{x \in X} (G^{'}(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \leq \bigvee_{\mathcal{V} \in \in^{(\mathcal{U})}} \bigwedge_{x \in X} (G^{'}(x) \vee \bigvee_{A \in \mathcal{V}} A(x))$$

Lemma 2.4([10]) Let L be complete Heyting algebra and let $f : (X, \delta_1) \to (Y, \delta_2)$ be a mapping. Then for any family $\mathcal{P} \subset L^Y$

$$\bigvee_{y \in Y} (f(G)(y) \wedge \bigwedge_{B \in \mathcal{P}} B(y)) = \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{P}} f^{-1}(B)(x))$$

Definition 2.5([10, 11]) Let (X, δ) be an $L-$space, $a \in L \backslash \{1\}$, and $G \in L^X$. A family $\mathcal{U} \subset L^X$ is said to be

(1) an $a-$shading of G if for any $x \in X, (G^{'}(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \nleq a$;

(2) a strong $a-$shading of G if $\bigwedge_{x \in X} (G^{'}(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \nleq a$;

(3) an $a-$ remote family $\mathcal{P}$ of G if for any $x \in X, (G(x) \wedge \bigwedge_{A \in \mathcal{U}} B(x)) \ngeq a$;

(4) a strong $a-$ remote family of G if $\bigvee_{x \in X} (G(x) \wedge \bigwedge_{A \in \mathcal{U}} A(x)) \ngeq a$.

Definition 2.6([10]) Let $a \in L \backslash \{0\}, G \in L^X$. A family $\mathcal{U} \subset L^X$ is called

(1) a β_a- cover of G if for all $x \in X$, it follows that $a \in \beta(G^{'}(x) \vee \bigvee\limits_{A \in \mathcal{U}} A(x))$;

(2) a strong β_a- cover of G if $a \in \beta(\bigwedge\limits_{x \in X}(G^{'}(x) \vee \bigvee\limits_{A \in \mathcal{U}} A(x)))$;

(3) a Q_a- cover of G if for all $x \in X$, it follows that $G^{'}(x) \vee \bigvee\limits_{A \in \mathcal{U}} A(x) \geq a$.

Definition 2.7([1, 9]) Let (X, δ_1) and (Y, δ_2) be two $L-$spaces. $f : (X, \delta_1) \to (Y, \delta_2)$ is called

(1) semicontinuous if $f^{-1}(A)$ is semiopen in (X, δ_1) for every open $L-$set A in (Y, δ_2).

(2) irresolute if $f^{-1}(A)$ is semiopen in (X, δ_1) for every semiopen $L-$set A in (Y, δ_2).

3. Definition and characterizations of S^* closedness

Definition 3.1 Let (X, δ) be an $L-$space. $G \in L^X$ is called S^* closedness if for every family $\mathcal{U}$ of semiopen $L-$ sets, it follows that

$$\bigwedge_{x \in X}(G^{'}(x) \vee \bigvee_{A \in \mathcal{U}} A(x)) \leq \bigvee_{\mathcal{V} \in \in^{(\mathcal{U})}} \bigwedge_{x \in X}(G^{'}(x) \vee \bigvee_{A \in \mathcal{V}} cl_* A(x))$$

Example 3.2 Suppose $X = \{x\}$, and $L = \{0, 1/4, 1/2, 3/4, 1\}$. For $a \in L$, $a^{'} = 1 - a$. Let $\delta = \{\underline{0}, A, \underline{1}\}$, where $A(x) = 3/4$, then (X, δ) is an $L-$space. We can see that every $L-$set in (X, δ) is S^* closedness.

Since any open $L-$set must be semiopen $L-$ set,we have the following theorem.

Theorem 3.3 Compactness is S^* closedness in $L-$space.

From Definition 3.1 and quasicomplementation of L, we can obtain the following theorem.

Theorem 3.4 Let (X, δ) be an $L-$space. $G \in L^X$ is S^* closedness if and only for every family $\mathcal{B}$ of semiclosed $L-$ sets, it follows that

$$\bigvee_{x \in X}(G(x) \wedge \bigwedge_{B \in \mathcal{B}} B(x)) \geq \bigwedge_{\mathcal{F} \in \in^{(\mathcal{B})}} \bigvee_{x \in X}(G(x) \wedge \bigwedge_{B \in \mathcal{F}} int_* B(x))$$

Definition 3.5 Let $a \in L \backslash \{0\}, G \in L^X$. A family $\mathcal{U} \subset L^X$ is said to have weak $a-$nonempty intersection in G if $\bigvee\limits_{x \in X}(G(x) \wedge \bigwedge\limits_{A \in \mathcal{U}} A(x)) \geq a$. $\mathcal{U}$ is said to have finite weak a_*-intersection property in G if for every finite subfamily $\mathcal{F}$ of $\mathcal{U}$, the family $\mathcal{F}_* = \{int_* F | F \in \mathcal{F}\}$ has weak $a-$nonempty intersection in G.

Theorem 3.6 Let (X, δ) be an $L-$space, $G \in L^X$. Then the following conditions are equivalent:

(1) G is S^* closedness.

(2) For any $a \in L \backslash \{1\}$, each semiopen strong $a-$ shading $\mathcal{U}$ of G has a finite subfamily $\mathcal{V}$ such that $\{cl_* A | A \in \mathcal{V}\}$ is a strong $a-$ shading of G .

(3) For any $a \in L \backslash \{0\}$, each semiclosed strong $a-$ remote family $\mathcal{P}$ of G has a finite subfamily $\mathcal{F}$ such that $\{int_* A | A \in \mathcal{P}\}$ is a strong $a-$ remote family of G.

(4) For any $a \in L \backslash \{0\}$, each family of semiclosed $L-$sets which has the finite weak a_*-intersection property in G has weak $a-$nonempty intersection in G.

Proof The proofs follow from Definition 3.1 and Theorem 3.4.

4. Properties of S^* closedness

Theorem 4.1 Let L be complete Heyting algebra. If G and H are S^* closedness, then so is $G \bigvee H$.

Proof For any family $\mathcal{P}$ of of semiclosed $L-$ sets, by Theorem 3.4 we have that

$$
\begin{aligned}
&\bigvee_{x \in X} ((G \bigvee H)(x) \wedge \bigwedge_{B \in \mathcal{P}} B(x)) \\
&\quad = \{ \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{P}} B(x))\} \bigvee \{ \bigvee_{x \in X} (H(x) \wedge \bigwedge_{B \in \mathcal{P}} B(x))\} \\
&\quad \geq \{ \bigwedge_{\mathcal{F} \in \in^{(\mathcal{P})}} \bigvee_{x \in X} (G(x) \wedge \bigwedge_{B \in \mathcal{F}} int_* B(x))\} \bigvee \{ \bigwedge_{\mathcal{F} \in \in^{(\mathcal{P})}} \bigvee_{x \in X} (H(x) \wedge \bigwedge_{B \in \mathcal{F}} int_* B(x))\} \\
&\quad = \bigwedge_{\mathcal{F} \in \in^{(\mathcal{P})}} \bigvee_{x \in X} ((G \bigvee H)(x) \wedge \bigwedge_{B \in \mathcal{F}} int_* B(x))
\end{aligned}
$$

This shows that $G \bigvee H$ is S^* closedness.

Theorem 4.2 If G is S^* closedness and H is a semiclosed set, then $G \bigwedge H$ is S^* closedness.

Proof For any family $\mathcal{P}$ of of semiclosed $L-$ sets, by Theorem 3.4 we have that

$$
\begin{aligned}
&\bigvee_{x\in X}((G\bigwedge H)(x)\wedge\bigwedge_{B\in\mathcal{P}}B(x))\\
&\quad=\bigvee_{x\in X}(G(x)\wedge\bigwedge_{B\in\mathcal{P}\bigcup\{\mathcal{H}\}}B(x))\\
&\quad\geq\{\bigwedge_{\mathcal{F}\in\in^{(\mathcal{P}\bigcup\{\mathcal{H}\})}}\bigvee_{x\in X}(G(x)\wedge\bigwedge_{B\in\mathcal{F}}int_*B(x))\}\\
&\quad=\{\bigwedge_{\mathcal{F}\in\in^{(\mathcal{P})}}\bigvee_{x\in X}(G(x)\wedge\bigwedge_{B\in\mathcal{F}}int_*B(x))\}\\
&\quad\bigvee\{\bigwedge_{\mathcal{F}\in\in^{(\mathcal{P})}}\bigvee_{x\in X}(G(x)\bigwedge H(x)\wedge\bigwedge_{B\in\mathcal{F}}int_*B(x))\}\\
&\quad=\bigwedge_{\mathcal{F}\in\in^{(\mathcal{P})}}\bigvee_{x\in X}((G\bigwedge H)(x)\wedge\bigwedge_{B\in\mathcal{F}}int_*B(x))
\end{aligned}
$$

Then we obtain that $G\bigwedge H$ is S^* closedness.

Theorem 4.3 Let L be complete Heyting algebra and $f:(X,\delta_1)\to(Y,\delta_2)$ be an irresolute mapping. If G is S^* closedness in (X,δ_1), then $f(G)$ is S^* closedness in (Y,δ_2).

Proof For any family $\mathcal{P}$ of of semiclosed $L-$ sets in (Y,δ_2), by Lemma 2.4 and S^* closedness of G we have that

$$
\begin{aligned}
&\bigvee_{y\in Y}((f(G))(y)\wedge\bigwedge_{B\in\mathcal{P}}B(y))\\
&\quad=\bigvee_{x\in X}(G(x)\wedge\bigwedge_{B\in\mathcal{P}}f^{-1}(B)(x))\\
&\quad\geq\bigwedge_{\mathcal{F}\in\in^{(\mathcal{P})}}\bigvee_{x\in X}(G(x)\wedge\bigwedge_{B\in\mathcal{F}}int_*f^{-1}(B)(x))\\
&\quad\geq\bigwedge_{\mathcal{F}\in\in^{(\mathcal{P})}}\bigvee_{x\in X}(G(x)\wedge\bigwedge_{B\in\mathcal{F}}f^{-1}(int_*B)(x))\\
&\quad=\bigwedge_{\mathcal{F}\in\in^{(\mathcal{P})}}\bigvee_{y\in Y}(f(G)(y)\wedge\bigwedge_{B\in\mathcal{F}}int_*B(y))
\end{aligned}
$$

Hence $f(G)$ is S^* closedness.

Analogously, we can obtain the following theorem.

Theorem 4.4 Let L be complete Heyting algebra and $f:(X,\delta_1)\to(Y,\delta_2)$ be a semicontinous mapping. If G is S^* closedness in (X,δ_1), then $f(G)$ is S^* closedness in (Y,δ_2).

5. Further characterizations of S^* closedness and goodness

In this section, we assume that L is a completely distributive $DeMorgan$ algebra.

Theorem 5.1 Let (X, δ) be an $L-$space, $G \in L^X$. Then the following statements are equivalent:

(1) G is S^* closedness.

(2) For any $a \in L\backslash\{0\}$, each semiclosed strong $a-$ remote family $\mathcal{P}$ of G has a finite subfamily $\mathcal{F}$ such that $\{int_*A|A \in \mathcal{F}\}$ is a strong $a-$ remote family of G.

(3) For any $a \in L\backslash\{0\}$, each semiclosed strong $a-$ remote family $\mathcal{P}$ of G has a finite subfamily $\mathcal{F}$ such that $\{int_*A|A \in \mathcal{F}\}$ is an $a-$ remote family of G.

(4) For any $a \in L\backslash\{0\}$, each semiclosed strong $a-$ remote family $\mathcal{P}$ of G there exist a finite subfamily $\mathcal{F}$ of $\mathcal{P}$ and $b \in \beta(a)$ such that $\{int_*A|A \in \mathcal{F}\}$ is a strong $b-$ remote family of G.

(5) For any $a \in L\backslash\{0\}$, each semiclosed strong $a-$ remote family $\mathcal{P}$ of G there exist a finite subfamily $\mathcal{F}$ of $\mathcal{P}$ and $b \in \beta(a)$ such that $\{int_*A|A \in \mathcal{F}\}$ is a $b-$ remote family of G.

(6) For any $a \in L\backslash\{1\}$, each semiopen strong $a-$ shading $\mathcal{U}$ of G has a finite subfamily $\mathcal{V}$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a strong $a-$ shading of G .

(7) For any $a \in L\backslash\{1\}$, each semiopen strong $a-$ shading $\mathcal{U}$ of G has a finite subfamily $\mathcal{V}$ such that $\{cl_*A|A \in \mathcal{V}\}$ is an $a-$ shading of G .

(8) For any $a \in L\backslash\{1\}$ and any semiopen strong $a-$ shading $\mathcal{U}$ of G ,there exist a finite subfamily $\mathcal{V}$ and $b \in \alpha(a)$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a $b-$ shading of G .

(9) For any $a \in L\backslash\{1\}$ and any semiopen strong $a-$ shading $\mathcal{U}$ of G ,there exist a finite subfamily $\mathcal{V}$ and $b \in \alpha(a)$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a strong $b-$ shading of G .

(10) For any $a \in L\backslash\{0\}$, each semiopen strong β_a-cover $\mathcal{U}$ of G has a finite subfamily $\mathcal{V}$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a strong β_a-cover of G.

(11) For any $a \in L\backslash\{0\}$, each semiopen strong β_a-cover $\mathcal{U}$ of G has a finite subfamily $\mathcal{V}$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a β_a-cover of G.

(12) For any $a \in L\backslash\{0\}$ and any semiopen strong β_a-cover $\mathcal{U}$ of G, there exist a finite subfamily $\mathcal{V}$ and $b \in L$ with $a \in \beta(b)$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a strong β_a-cover of G.

(13) For any $a \in L\backslash\{0\}$ and any semiopen strong β_a-cover $\mathcal{U}$ of G, there exist a finite subfamily $\mathcal{V}$ and $b \in L$ with $a \in \beta(b)$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a β_a-cover of G.

(14) For any $a \in L\backslash\{0\}$ and $b \in \beta(a)\backslash\{0\}$,each semiopen Q_a-cover $\mathcal{U}$ of G has a finite subfamily $\mathcal{V}$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a Q_b-cover of G.

(15) For any $a \in L\backslash\{0\}$ and $b \in \beta(a)\backslash\{0\}$, each semiopen Q_a-cover $\mathcal{U}$ of G has a finite subfamily $\mathcal{V}$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a strong β_b-cover of G.

(16) For any $a \in L\backslash\{0\}$ and $b \in \beta(a)\backslash\{0\}$, each semiopen Q_a-cover $\mathcal{U}$ of G has a finite subfamily $\mathcal{V}$ such that $\{cl_*A|A \in \mathcal{V}\}$ is a β_b-cover of G.

Proof (1) $\Leftrightarrow$ (2) It follows from Theorem 3.6.

(2) $\Rightarrow$ (3) It is easy for the fact that every strong $a-$ remote family of G is $a-$ remote family of G.

(3) $\Rightarrow$ (4) Let $a \in L\backslash\{0\}$. Suppose $\mathcal{U}$ is semiclosed strong $a-$ remote family of G, then $\bigvee\limits_{x\in X}(G(x) \wedge \bigwedge\limits_{A\in\mathcal{U}} A(x)) \not\geq a$. Take $c \in \beta(a)$ such that $\bigvee\limits_{x\in X}(G(x) \wedge \bigwedge\limits_{A\in\mathcal{U}} A(x)) \not\geq c$. One can see that $\mathcal{U}$ is semiclosed strong $c-$ remote family of G. From (3), $\mathcal{U}$ has a finite subfamily $\mathcal{F}$ such that $\{int_*A|A \in \mathcal{F}\}$ is $c-$ remote family of G. Choose $b \in \beta(a)$ such that $c \in \beta(b)$, then $\{int_*A|A \in \mathcal{F}\}$ is strong $b-$ remote family of G.

(4) $\Rightarrow$ (5) $\Rightarrow$ (2) It is Obvious.

(1) $\Leftrightarrow$ (6) It follows from Theorem 3.6.

(6) $\Rightarrow$ (7) It is easy for the fact that every strong $a-$shading family of G is $a-$shading family of G.

(7) $\Rightarrow$ (8)Let $a \in L\backslash\{1\}$. Suppose $\mathcal{U}$ is semiclosed strong $a-$ shading family of G, then $\bigwedge\limits_{x\in X}(G^{'}(x) \vee \bigvee\limits_{A\in\mathcal{U}} A(x)) \not\leq a$. Take $c \in \alpha(a)$ such that $\bigwedge\limits_{x\in X}(G^{'}(x) \vee \bigvee\limits_{A\in\mathcal{U}} A(x)) \not\leq c$. One can see that $\mathcal{U}$ is semiclosed strong $c-$shading family of G. From (7),$\mathcal{U}$ has a finite subfamily $\mathcal{F}$ such that $\{int_*A|A \in \mathcal{F}\}$ is $c-$shading family of G. Choose $b \in \alpha(a)$ such that $c \in \alpha(b)$, then $\{int_*A|A \in \mathcal{F}\}$ is strong $b-$shading family of G.

(8) $\Rightarrow$ (9) $\Rightarrow$ (6) It is Obvious.

Similarly we can prove the other statements.

Remark 5.2 In Theorem 5.1, $a \in L\backslash\{0\}$ and $b \in \beta(a)$ can be replaced by $a \in M(L)$ and $b \in \beta^*(a)$respectively. $a \in L\backslash\{1\}$ and $b \in \alpha(a)$ can be replaced by $a \in P(L)$ and $b \in \alpha^*(a)$ respectively. Thus, we can obtain other 15 equivalent conditions of the S^* closedness.

Lemma 5.3([11]) Let $(X, \omega(\delta))$ be generated topologically by (X, δ). If A is a semiopen set in (X, δ), then χ_A is a semiopen set $L-$ set in $(X, \omega(\delta))$. If B is a semiopen set in $(X, \omega(\delta))$, then $B_{(a)}$ is a semiopen set in (X, δ).

Lemma 5.4 Let $(X,\omega(\delta))$ be generated topologically by (X,δ),$B \in L^X$. Then $cl_*(B_{[b]}) \subset (cl_*B)_{[b]}$ in $(X,\omega(\delta))$.

Proof For $B \in L^X$, $B_{[b]} \subset (cl_*B)_{[b]}$. Then $cl_*(B_{[b]}) \subset (cl_*B)_{[b]}$ in $(X,\omega(\delta))$ since $(cl_*B)_{[b]}$ is a semiclosed set in $(X,\omega(\delta))$ and $cl_*(B_{[b]})$ is the smallest semiclosed set contains $B_{[b]}$.

Theorem 5.5 Let (X,δ) be a topological space and $(X,\omega(\delta))$ be generated topologically by (X,δ). Then $(X,\omega(\delta))$ is S^* closedness if and only if (X,δ) is S^* closedness.

Proof Let $\mathcal{A}$ be a semiopen-cover of (X,δ). Then $\{\chi_A|A\in\mathcal{A}\}$ is a family of semiopen $L-$ sets in $(X,\omega(\delta))$ with $\bigwedge\limits_{x\in X}(\bigvee\limits_{A\in\mathcal{U}}\chi_A(x))=\underline{1}$. From the S^* closedness of $(X,\omega(\delta))$, it follows that

$\underline{1}=\bigvee\limits_{\mathcal{V}\in\in^{(\mathcal{U})}}\bigwedge\limits_{x\in X}(\bigvee\limits_{A\in\mathcal{V}}cl_*(\chi_A)(x))\leq\bigvee\limits_{\mathcal{V}\in\in^{(\mathcal{U})}}\bigwedge\limits_{x\in X}(\bigvee\limits_{A\in\mathcal{V}}\chi_{cl_*A}(x))$

This implies that there exists $\mathcal{V}\in\in^{(\mathcal{U})}$ such that $\bigvee\limits_{A\in\mathcal{V}}\chi_{cl_*A}(x)=\underline{1}$. Hence, the family of $\{cl_*(A)|A\in\mathcal{V}\}$ is a cover of (X,δ). Then (X,δ) is S^* closedness.

Conversely, let $\mathcal{U}$ be a family of semiopen $L-$ sets in $(X,\omega(\delta))$ and let $\bigwedge\limits_{x\in X}(\bigvee\limits_{B\in\mathcal{U}}B(x))=a$. If $a=0$, then obviously

$$\bigwedge_{x\in X}(\bigvee_{B\in\mathcal{U}}B(x))\leq\bigvee_{\mathcal{V}\in\in^{(\mathcal{U})}}\bigwedge_{x\in X}(\bigvee_{B\in\mathcal{V}}cl_*B(x))$$

Now we suppose that $a\neq 0$. In this case, for any $b\in\beta(a)\backslash\{0\}$, we have that

$$b\in\beta(\bigwedge_{x\in X}(\bigvee_{B\in\mathcal{U}}B(x)))\subset\bigcap_{x\in X}\beta(\bigvee_{B\in\mathcal{U}}B(x))=\bigcap_{x\in X}\bigcup_{B\in\mathcal{U}}\beta(B(x))$$

This implies the family of $\{B_{(b)}|B\in\mathcal{U}\}$ is a semiopen cover of (X,δ) by Lemma 5.3, From the S^* closedness of (X,δ), there exists $\mathcal{V}\in\in^{(\mathcal{U})}$ such that $\{cl_*(B_{(b)})|B\in\mathcal{V}\}$ is a cover of (X,δ). Obviously $\{(cl_*B)_{[b]}|B\in\mathcal{V}\}$ is a cover of (X,δ) since $cl_*(B_{(b)})\subset cl_*(B_{[b]})\subset(cl_*B)_{[b]}$. Hence $b\leq\bigwedge\limits_{x\in X}(\bigvee\limits_{B\in\mathcal{V}}cl_*B(x))$. Further we have that

$$b\leq\bigwedge_{x\in X}(\bigvee_{B\in\mathcal{V}}cl_*B(x))\leq\bigvee_{\mathcal{V}\in\in^{(\mathcal{U})}}\bigwedge_{x\in X}(\bigvee_{B\in\mathcal{V}}cl_*B(x))$$

This implies that

$$\bigwedge_{x\in X}(\bigvee_{B\in\mathcal{U}}B(x))=a=\bigvee\{b|b\in\beta(a)\}\leq\bigvee_{\mathcal{V}\in\in^{(\mathcal{U})}}\bigwedge_{x\in X}(\bigvee_{B\in\mathcal{V}}cl_*B(x))$$

Therefore $(X, \omega(\delta))$ is S^* closedness.

Corollary 5.6 S^* closedness in $L-$topological spaces is good extension.

References

[1] K. K. Azad, On fuzzy semicontinuity,fuzzy almost continuity and fuzzy weakly continuity, J. Math. Anal. Appl, 82, pp. 14-32, (1981).

[2] C. L. Chang, Fuzzy topological spaces, J. Math. Anal. Appl, 24, pp. 182-190, (1968).

[3] B. Ghosh, Semicontinuous and semiclosed mappings and semi-connectedness in fuzzy setting, Fuzzy Sets and Systems, 35, pp. 345-355, (1990).

[4] G. Gierz, et al., A compendium of continuous Lattices, Springer Verlag, Berlin,1980.

[5] S. R. T. Kudri, Semicompactness and S^*-closedness in $L-$fuzzy topological spaces, Fuzzy Sets and Systems, 109, pp. 223-231, (2000).

[6] S. R. T. Kudri, M. W. Warner, Some good $L-$fuzzy compactness-related concepts and their properties I, Fuzzy Sets and Systems, 76, pp. 141-155, (1995).

[7] S. R. T. Kudri, M. W. Warner, Some good $L-$fuzzy compactness-related concepts and their properties II, Fuzzy Sets and Systems, 76, pp. 157-168, (1995).

[8] Y. M. Liu, M. K. Luo, Fuzzy topology,World Scientific,Singapore, (1997).

[9] S. Malakar, On fuzzy semi-irresolute and strongly irresolute functions, Fuzzy Sets and Systems, 45, pp. 239-244, (1992).

[10] F. G. Shi, A new notion of fuzzy compactness in $L-$topological spaces, Information Science. 173, pp. 35-48, (2005).

[11] F. G. Shi, Semicompactness in $L-$topological spaces, Internatinal Journal of Mathematics and Mathematical Sciences, 12, pp. 1869-1878, (2005).

[12] F. G. Shi, A new form of fuzzy $\beta-$compactness, Proyecciones Journal of Mathematics, 24, pp. 105-119, (2005).

[13] F. G. Shi, Countable compactness and the Lindelöf property of $L-$fuzzy sets, Iranian Journal of Fuzzy System, 1, pp. 79-88, (2004).

[14] F. G. Shi, A new approach to fuzzy almost compactness, Proyecciones Journal of Mathematics, 28, pp. 75-87, (2009).

[15] F. G. Shi, A new form of fuzzy $\alpha-$compactness, Mathematica Bohemica 131, pp. 15-28, (2006).

[16] G. J. Wang, Theory of $L-$fuzzy topological spaces,Shaanxi Normal University Press, Xi'an, (1988).

Bo Chen
School of Mathematics and Statistics
Southwest University
Chongqing 400715
P. R. China
e-mail : bobochen@swu.edu.cn