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SEQUENTIAL S^*-COMPACTNESS IN L-TOPOLOGICAL SPACES *

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Abstract

In this paper, a new notion of sequential compactness is introduced in L-topological spaces, which is called sequentially S^*-compactness. If $L = [0,1]$, sequential ultra-compactness, sequential N-compactness and sequential strong compactness imply sequential S^*-compactness, and sequential S^*-compactness implies sequential F-compactness. The intersection of a sequentially S^*-compact L-set and a closed L-set is sequentially S^*-compact. The continuous image of an sequentially S^*-compact L-set is sequentially S^*-compact. A weakly induced L-space $(X, \mathcal{T})$ is sequentially S^*-compact if and only if $(X, [\mathcal{T}])$ is sequential compact. The countable product of sequential S^*-compact L-sets is sequentially S^*-compact.

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1. Introduction and Preliminaries

In [6], the notion of S^*-compactness was presented in L-topological spaces by means of β_a-cover and Q_a-cover. It can be characterized in terms of constant a-nets and their weak O-cluster points. If $L = [0, 1]$, then strong compactness implies S^*-compactness and S^*-compactness implies Lowen's compactness.

In this paper, based on the notion of S^*-compactness, we shall introduce the notion of sequential S^*-compactness by constant a-sequences and their weak O-limit points. We shall research its properties and the relation between it and other sequential compactness.

Throughout this paper $(L, \bigvee, \bigwedge, ')$ is a completely distributive de Morgan algebra, X is a nonempty set. L^X is the set of all L-fuzzy sets on X. The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$.

An element a in L is called prime if $a \geq b \wedge c$ implies $a \geq b$ or $a \geq c$. An element a in L is called co-prime if a' is a prime element [3]. The set of nonunit prime elements in L is denoted by $P(L)$. The set of nonzero co-prime elements in L is denoted by $M(L)$. The set of nonzero co-prime elements in L^X is denoted by $M(L^X)$.

The binary relation $\prec$ in L is defined as follows: for $a, b \in L$, $a \prec b$ if and only if for every subset $D \subseteq L$, the relation $b \leq \sup D$ always implies the existence of $d \in D$ with $a \leq d$ [2]. In a completely distributive de Morgan algebra L, each member b is a sup of $\{a \in L \mid a \prec b\}$. In the sense of [4, 9], $\{a \in L \mid a \prec b\}$ is the greatest minimal family of b, in symbol $\beta(b)$. Moreover for $b \in L$, define $\alpha(b) = \{a \in L \mid a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \cap P(L)$.

For an L-set $A \in L^X$, $\beta(A)$ denotes the greatest minimal family of A and $\beta^*(A) = \beta(A) \bigcap M(L^X)$.

For $a \in L$ and $A \in L^X$, we use the following notations in [6].

$$A_{(a)} = \{x \in X \mid a \in \beta(A(x))\}, \quad A^{(a)} = \{x \in X \mid A(x) \not\leq a\}.$$

An L-topological space (or L-space for short) is a pair $(X, \mathcal{T})$, where $\mathcal{T}$ is a subfamily of L^X which contains $\underline{0}$, $\underline{1}$ and is closed for any suprema and finite infima. $\mathcal{T}$ is called an L-topology on X. Each member of $\mathcal{T}$ is called an open L-set and its complement is called a closed L-set.

Definition 1.1 ([4, 9]). *For a topological space (X, τ), let $\omega_L(\tau)$ denote the family of all the lower semi-continuous maps from (X, τ) to L, i.e.,*

$\omega_L(\tau) = \{A \in L^X \mid A^{(a)} \in \tau, a \in L\}$. *Then* $\omega_L(\tau)$ *is an* L*-topology on* X, *in this case,* $(X, \omega_L(\tau))$ *is called topologically generated by* (X, τ).

Definition 1.2 ([4, 9]). *An L-space* $(X, \mathcal{T})$ *is called weak induced if* $\forall a \in L$, $\forall A \in \mathcal{T}$, *it follows that* $A^{(a)} \in [\mathcal{T}]$, *where* $[\mathcal{T}]$ *denotes the topology formed by all crisp sets in* $\mathcal{T}$.

Lemma 1.3 ([6]). *Let* $(X, \mathcal{T})$ *be a weakly induced L-space,* $a \in L, A \in \mathcal{T}$. *Then* $A_{(a)}$ *is an open set in* $[\mathcal{T}]$.

Ultra-sequential compactness, N-sequential compactness, strong sequential compactness and sequential compactness had be generalized to $[0,1]$-topological space by L.X. Xuan in [10, 11, 12, 13].

Definition 1.4 ([10]). *A* $[0,1]$*-space* $(X, \mathcal{T})$ *is called sequentially ultra-compact or ultra-sequentially compact if* $\iota_L(\mathcal{T})$ *is sequentially compact, where* $\iota(\mathcal{T})$ *is the topology generated by* $\{A^{(a)} \mid A \in \mathcal{T}, a \in [0,1]\}$.

Definition 1.5 ([4, 9]). *Let* $a \in M(L)$. *A net* $\{S(n) \mid n \in D\}$ *in* L^X *is said to be constant a-net if the height* $V(S(n))$ *of each* $S(n)$ *is a constant value* a. $\{S(n) \mid n \in D\}$ *is said to be an a-net if for each* $r \in \beta^*(a)$, *there exists* $n(r) \in D$ *such that* $V(S(n)) \geq r$ *for all* $n \geq n(r)$.

Definition 1.6 ([7]). *A net* S *with index set* D *in* L^X *is also denoted by* $\{S(n) \mid n \in D\}$ *or* $\{S(n)\}_{n \in D}$. *For* $G \in L^X$, *a net* S *is said to quasi-coincide with* G *if* $\forall n \in D, S(n) \not\leq G'$.

Definition 1.7 ([4, 9]). *Let* $\{S(n) \mid n \in D\}$ *be a net in* $(X, \mathcal{T})$, $x_\lambda \in M(L^X)$. x_λ *is called a cluster point of* S *if for each closed R-neighborhood* P *of* x_λ, S *is not frequently in* P. x_λ *is called a limit point of* S *if for each closed R-neighborhood* P *of* x_λ, S *is not eventually in* P, *in this case we also say that* S *converges to* x_λ.

Definition 1.8 ([11]). *A $[0,1]$-space $(X, \mathcal{T})$ is called sequentially N-compact or N-sequentially compact if for all $a \in (0,1]$, each a-sequence has a cluster point x_a.*

Definition 1.9 ([12]). *A $[0,1]$-space $(X, \mathcal{T})$ is called sequentially strongly compact or strongly sequentially compact if for all $a \in (0,1]$, each constant a-sequence has a cluster point x_a.*

Definition 1.10 ([13]). *A $[0,1]$-space $(X, \mathcal{T})$ is called sequentially F-compact or fuzzy sequentially compact if for all $a \in (0,1]$ and all $b \in (0,a)$, each constant a-sequence has a cluster point x_b.*

Definition 1.11 ([6]). *Let $(X, \mathcal{T})$ be an L-space. An open L-set U is called a strong open neighborhood of a fuzzy point x_λ, if $\lambda \in \beta(U(x))$.*

Definition 1.12 ([6]). *Let $\{S(n) \mid n \in D\}$ be a net in $(X, \mathcal{T})$, $x_\lambda \in M(L^X)$. x_λ is called a weak O-cluster point of S, if for each strong open neighborhood U of x_λ, S is frequently in U. x_λ is called a weak O-limit point of S, if for each strong open neighborhood U of x_λ, S is eventually in U, in this case we also say that S weakly O-converges to x_λ, denoted by $S \xrightarrow{WO} x_\lambda$.*

2. Sequentially S^*-compactness

Definition 2.1. *An L-set G in $(X, \mathcal{T})$ is called sequentially S^*-compact if for all $a \in M(L)$, each constant $a-$sequence quasi-coinciding with G has a subsequence which weak O-converges to $x_a \notin \beta(G')$*

Theorem 2.2. *An L-set with finite support is sequentially S^*-compact.*

Proof. Suppose that G is an L-set with finite support in $(X, \mathcal{T})$ and $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a-sequence quasi-coinciding with G. Then $\{x_a^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ such that $x^{n(k)} = x^{n(1)}$ for all $k \in \mathbf{N}$. In this case, $x_a^{n(1)}$ is a weak O-limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ and $x_a^{n(1)} \notin \beta(G')$

Definition 2.3. *An L-space $(X, \mathcal{T})$ is said to be strong C_I, if for each $x_\lambda \in M(L^X)$, there exists a countable local base of strong open neighborhoods of x_λ.*

It is easy to prove the following lemma.

Lemma 2.4. *An L-space $(X, \mathcal{T})$ is strong C_I if and only if for each $x_\lambda \in M(L^X)$, there exists a countable local base $\{U_i \mid i \in \mathbf{N}\}$ of strong open neighborhoods of x_λ such that $U_1 \supset U_2 \supset \cdots$.*

Theorem 2.5. *Let $(X, \mathcal{T})$ be a strong C_I L-space. Then $G \in L^X$ is sequentially S^*-compact if and only if each constant a-net quasi-coinciding with G has a weak O-cluster point $x_a \notin \beta(G')$.*

Proof. The necessity is obvious. Now we prove the sufficiency. Suppose that $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a-sequence quasi-coinciding with G and it has a weak O-cluster point $x_a \notin \beta(G')$. Let $\{U_i \mid i \in \mathbf{N}\}$ be a countable local base at x_a such that $U_1 \supset U_2 \supset \cdots$. Then for each $k \in \mathbf{N}$, there exists an $n(k) \geq k$ such that $x_a^{n(k)} \leq U_k$. It is easy to see that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converges to x_a. Therefore $G \in L^X$ is sequentially S^*-compact.

By Theorem 2.5 and analogous to the proof of Theorem 5.3 in [6], we can obtain the following result.

Theorem 2.6. *Let $(X, \mathcal{T})$ be a C_{II} L-space. Then $G \in L^X$ is sequentially S^*-compact if and only if it is S^*-compact.*

Theorem 2.7. *If G is sequentially S^*-compact and H is closed, then $G \wedge H$ is sequentially S^*-compact.*

Proof. Suppose that $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a-sequence quasi-coinciding with $G \wedge H$. Then $\{x_a^n \mid n \in \mathbf{N}\}$ is also quasi-coincident with G. By sequential S^*-compactness of G we know that $\{x_a^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converging to $x_a \notin \beta(G')$. We can prove that $x_a \notin \beta(H')$ since $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ is quasi-coincident with H. Therefore $x_a \notin \beta(G' \vee H')$.

Theorem 2.8. *If G is sequentially S^*-compact in $(X, \mathcal{T}_1)$ and $f : (X, \mathcal{T}_1) \to (Y, \mathcal{T}_2)$ is continuous, then $f_L^{\rightarrow}(G)$ is sequentially S^*-compact in $(Y, \mathcal{T}_2)$.*

Proof. Suppose that $\{y_a^n \mid n \in \mathbf{N}\}$ is a constant a-sequence quasi-coinciding with $f_L^{\rightarrow}(G)$. Then there exists a constant a-sequence $\{x_a^n \mid n \in \mathbf{N}\}$ quasi-coinciding with G such that $f(x^n) = y^n$ for all $n \in \mathbf{N}$. By sequential S^*-compactness of G we know that $\{x_a^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converging to $x_a \notin \beta(G')$. It is easy to prove that $\{y_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converges to $f(x)_a$ and $f(x)_a \notin \beta\left(f(G)'\right)$. Therefore $f_L^{\rightarrow}(G)$ is sequentially S^*-compact.

Theorem 2.9. *If $(X, \mathcal{T})$ is a weakly induced L-space, then $(X, \mathcal{T})$ is sequentially S^*-compact if and only if $(X, [\mathcal{T}])$ is sequentially compact.*

Proof. Let $(X, [\mathcal{T}])$ be sequentially compact. For $a \in M(L)$, let $\{x_a^n \mid n \in \mathbf{N}\}$ be a constant a-sequence quasi-coinciding with $\underline{1}$ in $(X, \mathcal{T})$. Then $\{x^n \mid n \in \mathbf{N}\}$ is a sequence in $(X, [\mathcal{T}])$. By sequential compactness of $(X, [\mathcal{T}])$, we know that $\{x^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x^{n(k)} \mid k \in \mathbf{N}\}$ converging to x. Now we prove that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converges to x_a in $(X, \mathcal{T})$. In fact, let U is a strong open neighborhood of x_a. By Lemma 1.3 we know that $U_{(a)}$ is an open neighborhood of x in $[\mathcal{T}]$. Thus $\{x^{n(k)} \mid k \in \mathbf{N}\}$ is eventually in $U_{(a)}$. Hence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ is eventually in U. It is proved that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converges to x_a. This shows that $(X, \mathcal{T})$ is sequentially S^*-compact.

Conversely let $(X, \mathcal{T})$ be sequentially S^*-compact and $\{x^n \mid n \in \mathbf{N}\}$ be a sequence in $(X, [\mathcal{T}])$. Then for each $a \in \beta^*(1)$, $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a-sequence quasi-coinciding with $\underline{1}$ in $(X, \mathcal{T})$. By sequential S^*-compactness of $(X, \mathcal{T})$, we know that $\{x_a^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converging to x_a in $(X, \mathcal{T})$. It is easy to prove that $\{x^{n(k)} \mid k \in \mathbf{N}\}$ converges to x in $(X, [\mathcal{T}])$. Therefore $(X, [\mathcal{T}])$ is sequentially compact.

Corollary 2.10. *Let (X, τ) be a crisp topological space. Then $(X, \omega_L(\tau))$ is sequentially S^*-compact if and only if (X, τ) is sequentially compact.*

Theorem 2.11. *Let $(X, \mathcal{T})$ be the countable product of a family of L-space's $\{(X_i, \mathcal{T}_i)\}_{i \in \mathbf{N}}$. If for each $i \in \mathbf{N}$, G_i is sequentially S^*-compact in $(X_i, \mathcal{T}_i)$, then $G = \prod_{i \in \mathbf{N}} G_i$ is sequentially S^*-compact in $(X, \mathcal{T})$.*

Proof. Let p_n denote the projection from X onto X_n. For $a \in M(L)$, let $\{x(i)_a \mid i \in \mathbf{N}\}$ be a constant a-sequence quasi-coinciding with G. Then for any $n \in \mathbf{N}$, $\{p_n(x(i))_a \mid i \in \mathbf{N}\}$ is a constant a-sequence quasi-coinciding with G_n. By sequential S^*-compactness of G_1, we can choose a subsequence $\{x^1(i)_a \mid i \in \mathbf{N}\}$ of $\{x(i)_a \mid i \in \mathbf{N}\}$ such that $\{p_1(x^1(i)_a) \mid i \in \mathbf{N}\}$ weak O-converges to a point $y_a^1 \notin \beta((G_1)')$. In this case $\{p_2(x^1(i)_a) \mid i \in \mathbf{N}\}$ is a constant a-sequence quasi-coinciding with G_2. By sequential S^*-compactness of G_2, we can again choose a subsequence $\{x^2(i)_a \mid i \in \mathbf{N}\}$ of $\{x^1(i)_a \mid i \in \mathbf{N}\}$ such that $\{p_2(x^2(i)_a) \mid i \in \mathbf{N}\}$ weak O-converges to a point $y_a^2 \notin \beta((G_2)')$. In this case $\{p_3(x^2(i)_a) \mid i \in \mathbf{N}\}$ is a constant a-sequence quasi-coinciding with G_3. $\cdots$, by reduction for all $k \in \mathbf{N}$, we can obtain a subsequence $\{x^{k+1}(i)_a \mid i \in \mathbf{N}\}$ of $\{x^k(i)_a \mid i \in \mathbf{N}\}$ such that $\{p_{k+1}(x^{k+1}(i)_a) \mid i \in \mathbf{N}\}$ weak O-converges to a point $y_a^{k+1} \notin \beta((G_{k+1})')$. Take a sequence $\{x^i(i)_a \mid i \in \mathbf{N}\}$. Then $\{x^i(i)_a \mid i \in \mathbf{N}\}$ is a subsequence of $\{x(i)_a \mid i \in \mathbf{N}\}$. It is easy to see that $\{x^i(i)_a \mid i \in \mathbf{N}\}$ becomes a subsequence of $\{x^k(i)_a \mid i \in \mathbf{N}\}$ after dropping first $k-1$ terms.

Let $y_a = \prod_{k \in \mathbf{N}} y_a^k$. Then $y_a \notin \beta(G')$. In fact, if $y_a \in \beta(G')$, then by

$$G = \prod_{n \in \mathbf{N}} G_n = \bigwedge_{n \in \mathbf{N}} p_n^{-1}(G_n)$$

we obtain that $y_a \in \beta\left(\bigvee_{n \in \mathbf{N}} p_n^{-1}(G_n)'\right)$, hence there is $n \in \mathbf{N}$ such that $y_a \in \beta\left(p_n^{-1}(G_n)'\right)$, this implies that $y_a^n \in \beta((G_n)')$, we obtain a contradiction.

Now we prove that y_a is a weak O-limit point of $\{x^i(i)_a \mid i \in \mathbf{N}\}$. Suppose that U is a strong open neighborhood of y_a in $\mathcal{T}$. Then there exists a finite subset T of $\mathbf{N}$ and open set $U_n \in \mathcal{T}_n$ for each $n \in T$ such that $\bigwedge_{n \in T} p_n^{-1}(U_n) \leq U$ and $y_a \in \beta\left(\bigwedge_{n \in T} p_n^{-1}(U_n)\right) \subset \beta(U)$. Hence for all $n \in T$, $y_a^n = p_n(y_a) \in \beta(U_n)$. Thus $\{p_n(x^i(i)_a) \mid i \in \mathbf{N}\}$ is eventually in U_n for each $n \in T$. This implies that $\{x^i(i)_a \mid i \in \mathbf{N}\}$ is eventually in $\bigwedge_{n \in T} p_n^{-1}(U_n) \leq U$. Therefore y_a is a weak O-limit point of $\{x^i(i)_a \mid i \in \mathbf{N}\}$. The proof is completed.

From Theorem 2.8 and Theorem 2.11 we can obtain the following corollary.

Corollary 2.12. *Let $(X, \mathcal{T})$ be the countable product of a family of L-space's $\{(X_i, \mathcal{T}_i)\}_{i \in \mathbf{N}}$. If for each $i \in \mathbf{N}$, $(X_i, \mathcal{T}_i)$ is sequentially S^*-compact, then so is $(X, \mathcal{T})$. Conversely if $(X, \mathcal{T})$ is sequentially S^*-compact and $(X_i, \mathcal{T}_i)$ is fully stratified, then $(X_i, \mathcal{T}_i)$ is sequentially S^*-compact.*

3. A comparison of a few kinds of sequential compactness

In order to compare different notions of fuzzy sequential compactness, we only consider sequential S^*-compactness of $[0, 1]$-topological spaces.

Theorem 3.1. *For a $[0, 1]$-space, sequential ultra-compactness, sequential N-compactness and sequential strong compactness imply sequential S^*-compactness, and sequential S^*-compactness implies sequential F-compactness.*

Proof. We only need to prove that sequential strong compactness implies sequential S^*-compactness and sequential S^*-compactness implies sequential F-compactness. Let $(X, \mathcal{T})$ be a sequentially strong compact $[0, 1]$-space. To prove that it is sequentially S^*-compact, suppose that $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a-sequence in $(X, \mathcal{T})$. If $a = 1$, then for all $x \in X$, x_1 can be regard as a weak O-limit point of $\{x_a^n \mid n \in \mathbf{N}\}$ since none of open fuzzy sets is strong open neighborhood of x_1. Now we suppose that $a \in (0, 1)$. Take $b = 1 - a$. Then $\{x_b^n \mid n \in \mathbf{N}\}$ is a constant b-sequence. By sequential strong compactness of $(X, \mathcal{T})$, we know that $\{x_b^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ converging to a point x_b. We shall prove that x_a is a weak O-limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$. In fact, for each strong open neighborhood U of x_a, U is an open quasi-neighborhood of x_b. Since $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ converges to x_b, we know that $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ is eventually quasi-coincident with U. This implies that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ is eventually in U. Therefore x_a is a weak O-limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$Sequential S^*-compactness of $(X, \mathcal{T})$ is proved.

Further let $(X, \mathcal{T})$ be a sequentially S^*-compact $[0, 1]$-space. To prove that it is sequentially F-compact, suppose that $\{x_a^n \mid n \in \mathbf{N}\}$ is a constant a-sequence, where $a \in (0, 1]$. For any $r \in (0, a)$, let $b = 1 - r$, then $\{x_b^n \mid n \in \mathbf{N}\}$ is a constant b-sequence. By sequential S^*-compactness of $(X, \mathcal{T})$, we know that $\{x_b^n \mid n \in \mathbf{N}\}$ has a subsequence $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converging to a point x_b. We shall prove that x_r is a limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$. In fact, for each open quasi-neighborhood U of x_r, U is a strong open neighborhood of x_b. Since $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ weak O-converges to x_b, we know that $\{x_b^{n(k)} \mid k \in \mathbf{N}\}$ is eventually in U. This implies that $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$ is eventually quasi-coincident with U. Therefore x_r is a limit point of $\{x_a^{n(k)} \mid k \in \mathbf{N}\}$. Sequential F-compactness of $(X, \mathcal{T})$ is proved.

In general, each inverse in Theorem 4.1 needn't be true. This can be seen from the following two examples.

Example 3.2. *Take $X = \mathbf{N}$. $\forall n \in X$, take $B_n \in [0,1]^X$ as follows:*

$$B_n(x) = \begin{cases} 0.5 + \dfrac{1}{n+1}, & x = n \\ 0.5 - \dfrac{1}{n+1}, & x \neq n \end{cases}$$

Let $\mathcal{T}$ be the [0,1]-topology generated by the subbase $\mathcal{B} = \{B_n \mid n \in \mathbf{N}\}$. As proved in [6], $(Y, \mathcal{T})$ is Lowen's fuzzy compact but it not S^*-compact. It is easy to see that $(Y, \mathcal{T})$ is a C_{II} [0,1]-space. Therefore it is sequentially F-compact but not sequentially S^*-compact.

Example 3.3. *Take $Y = \mathbf{N}$. $\forall n \in \mathbf{N}$, take $B_n \in [0,1]^Y$ as follows:*

$$B_n(y) = \begin{cases} \dfrac{1}{n+1} + 0.5, & y = n, \\ 0.5, & y \neq n. \end{cases}$$

Let $\mathcal{T}$ be the topology generated by the subbase $\mathcal{B} = \{B_n \mid n \in \mathbf{N}\}$. As proved in [6], $(Y, \mathcal{T})$ is S^*-compact but it not strong compact. It is easy to see that $(Y, \mathcal{T})$ is a C_{II} [0,1]-space. Therefore it is sequentially S^*-compact but not sequentially strong compact.

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