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Sum divisor cordial graphs

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Abstract

A sum divisor cordial labeling of a graph G with vertex set V is a bijection f from $V(G)$ to $\{1, 2, \cdots, |V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides $f(u) + f(v)$ and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that path, comb, star, complete bipartite, $K_2 + mK_1$, bistar, jewel, crown, flower, gear, subdivision of the star, $K_{1,3} * K_{1,n}$ and square graph of $B_{n,n}$ are sum divisor cordial graphs.

Subjclass : *05C78.*

Keywords : *Sum divisor cordial, divisor cordial.*

1. Introduction

All graphs considered here are simple, finite, connected and undirected. We follow the basic notations and terminologies of graph theory as in [2]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of vertices the labeling is called vertex labeling. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. For a dynamic survey of various graph labeling, we refer to Gallian [1].

Definition 1.1. *Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \to \{1, 2, \cdots, |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if either $f(u)|f(v)$ or $f(v)|f(u)$ and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a divisor cordial labeling is called a divisor cordial graph.*

Motivated by the concept of divisor cordial labeling, we introduce a new concept of divisor cordial labeling called sum divisor cordial labeling.

Definition 1.2. *Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \to \{1, 2, \cdots, |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if $2|(f(u) + f(v))$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.*

Definition 1.3. *The comb $P_n \odot K_1$ is the graph obtained from a path by attaching a pendant edge to each vertex of the path.*

Definition 1.4. *The bistar $B_{n,n}$ is the graph obtained by attaching the apex vertices of two copies of $K_{1,n}$ by an edge.*

Definition 1.5. *The complete bipartite graph is a simple bipartite graph such that every vertex in one of the bipartition subsets is joined to every vertex in the other bipartition subset. Any complete bipartite graph that has m vertices in one of its subsets and n vertices in other is denoted by $K_{n,m}$.*

Definition 1.6. *The join of two graphs G_1 and G_2 is denoted by $G_1 + G_2$ and whose vertex set is $V(G_1 + G_2) = V(G_1) \cup V(G_2)$ and edge set is $E(G_1 + G_2) = E(G_1) \cup E(G_2) \cup \{uv : u \in V(G_1), v \in V(G_2)\}$.*

Definition 1.7. *The jewel J_n is the graph with vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.*

Definition 1.8. *The crown $C_n \odot K_1$ is the graph obtained from a cycle by attaching a pendant edge to each vertex of the cycle.*

Definition 1.9. *The helm H_n is the graph obtained from a wheel by attaching a pendant edge to each rim vertex. The flower Fl_n is the graph obtained from a helm by attaching each pendant vertex to the apex of the helm.*

Definition 1.10. *The gear G_n is the graph obtained from a wheel by subdividing each of its rim edge.*

Definition 1.11. *$K_{1,3} * K_{1,n}$ is the graph obtained from $K_{1,3}$ by attaching root of a star $K_{1,n}$ at each pendant vertex of $K_{1,3}$.*

Definition 1.12. *For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 apart in G.*

Definition 1.13. *The subdivision of star $S(K_{1,n})$ is the graph obtained from $K_{1,n}$ by attaching a pendant edge to each vertex of $K_{1,n}$ except root vertex.*

2. main results

Theorem 2.1. *The path P_n is sum divisor cordial graph.*

Proof. Let P_n be a path with consecutive vertices $v_1, v_2, \cdots, v_n$. Then P_n is of order n and size $n-1$. Define $f : V(P_n) \to \{1, 2, \cdots, n\}$ as follows:

Case 1: n is odd

$$f(v_i) = \begin{cases} i & \text{if } i \equiv 0, 1 (mod\ 4) \\ i+1 & \text{if } i \equiv 2 (mod\ 4) \text{ for } 1 \leq i \leq n \\ i-1 & \text{if } i \equiv 3 (mod\ 4) \end{cases}$$

Case 2: n is even

$$f(v_i) = \begin{cases} i & \text{if } i \equiv 1, 2(mod\ 4) \\ i+1 & \text{if } i \equiv 3(mod\ 4) \text{ for } 1 \le i \le n \\ i-1 & \text{if } i \equiv 0(mod\ 4) \end{cases}$$

In both cases, the induced edge labels are

$$f^*(v_i v_{i+1}) = \begin{cases} 1 & \text{if } 2|(f(v_i) + f(v_{i+1})) \\ 0 & \text{otherwise} \end{cases}$$

We observe that,

$$e_f(0) = \begin{cases} \frac{n-1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

$$e_f(1) = \begin{cases} \frac{n-1}{2} & \text{if } n \text{ is odd} \\ \frac{n-2}{2} & \text{if } n \text{ is even} \end{cases}$$

Thus, $|e_f(0) - e_f(1)| \le 1$.
Hence, the path P_n is sum divisor cordial graph. □

Example 2.2. *A sum divisor cordial labeling of P_6 and P_7 is shown in Figure 2.1*

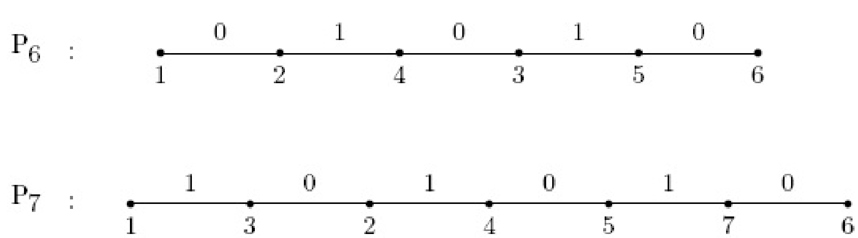

Figure 2.1

Theorem 2.3. *The comb is sum divisor cordial graph.*

Proof. Let G be a comb obtained from the path $v_1, v_2, \cdots, v_n$ by joining a vertex u_i to v_i for each $i = 1, 2, \cdots, n$. Then G is of order $2n$ and size $2n - 1$. Define $f : V(G) \to \{1, 2, \cdots, 2n\}$ as follows:

$$f(v_i) = 2i - 1; 1 \leq i \leq n$$
$$f(u_i) = 2i; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(v_i v_{i+1}) = 1; 1 \leq i \leq n - 1$$
$$f^*(v_i u_i) = 0; 1 \leq i \leq n$$

We observe that, $e_f(0) = n$ and $e_f(1) = n - 1$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the comb is sum divisor cordial graph. □

Example 2.4. *A sum divisor cordial labeling of comb is shown in Figure 2.2*

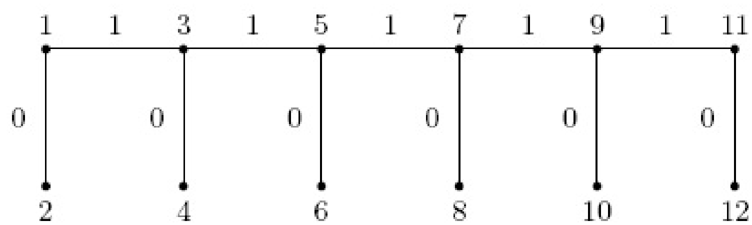

Figure 2.2

Theorem 2.5. *The star $K_{1,n}$ is sum divisor cordial graph.*

Proof. Let (V_1, V_2) be the bipartition of $K_{1,n}$ with $V_1 = \{u\}$ and $V_2 = \{u_1, u_2, \cdots, u_n\}$. Let $E(K_{1,n}) = \{uu_i : 1 \leq i \leq n\}$. Then $K_{1,n}$ is of order $n+1$ and size n. Define $f : V(K_{1,n}) \to \{1, 2, \cdots, n+1\}$ as follows:

$$f(u) = 1;$$
$$f(u_i) = i + 1; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(uu_{2i-1}) = 0; 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil$$

$$f^*(uu_{2i}) = 1; 1 \leq i \leq \left\lfloor \frac{n}{2} \right\rfloor$$

We observe that,

$$e_f(0) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

$$e_f(1) = \begin{cases} \frac{n-1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, the star $K_{1,n}$ is sum divisor cordial graph. □

Example 2.6. *A sum divisor cordial labeling of $K_{1,5}$ is shown in Figure 2.3.*

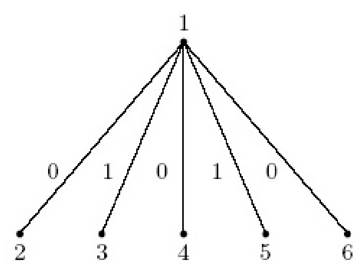

Figure 2.3

Theorem 2.7. *The graph $K_{2,n}$ is sum divisor cordial graph.*

Proof. Let (V_1, V_2) be the bipartition of $K_{2,n}$ with $V_1 = \{u, v\}$ and $V_2 = \{v_1, v_2, \cdots, v_n\}$. Let $E(K_{2,n}) = \{uv_i, vv_i : 1 \leq i \leq n\}$. Then $K_{2,n}$ is of order $n + 2$ and size $2n$. Define $f : V(K_{2,n}) \to \{1, 2, \cdots, n + 2\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(v_i) = i + 2; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(uv_{2i-1}) = 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(uv_{2i}) = 0; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(vv_{2i-1}) = 0; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$

$$f^*(vv_{2i}) = 1; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$$

We observe that, $e_f(0) = e_f(1) = n$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $K_{2,n}$ is sum divisor cordial graph. □

Example 2.8. *A sum divisor cordial labeling of $K_{2,5}$ is shown in Figure 2.4.*

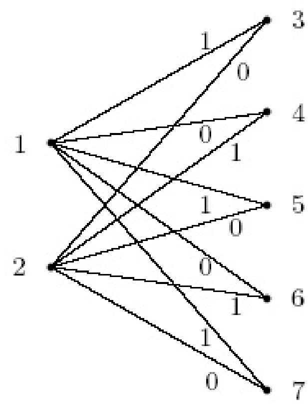

Figure 2.4

Theorem 2.9. *The graph $K_2 + mK_1$ is sum divisor cordial graph.*

Proof. Let $G = K_2 + mK_1$. Let $V(G) = \{u, v, w_1, w_2, \cdots, w_m\}$ and $E(G) = \{uv, uw_i, vw_i : 1 \leq i \leq m\}$. Then G is of order $m + 2$ and size $2m + 1$. Define $f : V(G) \to \{1, 2, \cdots, m + 2\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(w_i) = i + 2; 1 \leq i \leq m.$$

Then, the induced edge labels are

$$f^*(uv) = 0;$$
$$f^*(uw_{2i-1}) = 1; 1 \leq i \leq \lceil \frac{m}{2} \rceil$$
$$f^*(uw_{2i}) = 0; 1 \leq i \leq \lfloor \frac{m}{2} \rfloor$$
$$f^*(vw_{2i-1}) = 0; 1 \leq i \leq \lceil \frac{m}{2} \rceil$$
$$f^*(vw_{2i}) = 1; 1 \leq i \leq \lfloor \frac{m}{2} \rfloor$$

We observe that, $e_f(0) = m + 1$ and $e_f(1) = m$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $K_2 + mK_1$ is sum divisor cordial graph. □

Example 2.10. *A sum divisor cordial labeling of $K_2 + 5K_1$ is shown in Figure 2.5.*

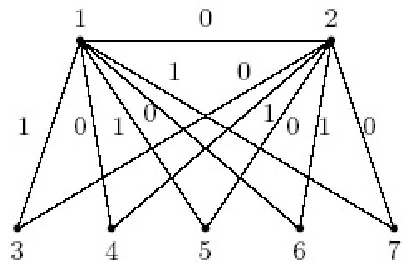

Figure 2.5

Theorem 2.11. *The bistar $B_{n,n}$ is sum divisor cordial graph.*

Proof. Let $G = B_{n,n}$. Let $V(G) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{uv, vv_i, uu_i : 1 \leq i \leq n\}$. Then G is of order $2n + 2$ and size $2n + 1$. Define $f : V(G) \to \{1, 2, \cdots, 2n + 2\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(u_{2i-1}) = 4i - 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f(u_{2i}) = 4i + 2; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f(v_{2i-1}) = 4i; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f(v_{2i}) = 4i + 1; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$

Then, the induced edge labels are

$$f^*(uv) = 0;$$
$$f^*(uu_{2i-1}) = 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(uu_{2i}) = 0; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(vv_{2i-1}) = 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(vv_{2i}) = 0; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$

We observe that,

$$e_f(0) = \begin{cases} n & \text{if } n \text{ is odd} \\ n + 1 & \text{if } n \text{ is even} \end{cases}$$

$$e_f(1) = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ n & \text{if } n \text{ is even} \end{cases}$$

Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, the bistar $B_{n,n}$ is sum divisor cordial graph. □

Example 2.12. *A sum divisor cordial labeling of $B_{5,5}$ is shown in Figure 2.6.*

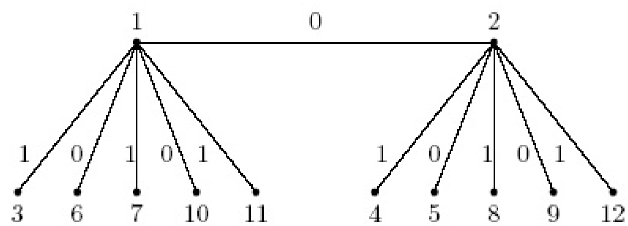

Figure 2.6

Theorem 2.13. *The flower Fl_n is sum divisor cordial graph.*

Proof. Let $G = Fl_n$. Let $V(G) = \{v, v_i, u_i : 1 \leq i \leq n\}$ and $E(G) = \{vv_i, v_iu_i, vu_i : 1 \leq i \leq n; v_nv_1; v_iv_{i+1} : 1 \leq i \leq n-1\}$. Then G is of order $2n+1$ and size $4n$. Define $f : V(G) \rightarrow \{1, 2, \cdots, 2n+1\}$ as follows:

$$f(v) = 1;$$
$$f(v_i) = 2i; 1 \leq i \leq n$$
$$f(u_i) = 2i+1; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(vv_i) = 0; 1 \leq i \leq n$$
$$f^*(vu_i) = 1; 1 \leq i \leq n$$
$$f^*(v_iu_i) = 0; 1 \leq i \leq n$$
$$f^*(v_iv_{i+1}) = 1; 1 \leq i \leq n-1$$
$$f^*(v_nv_1) = 1;$$

We observe that, $e_f(0) = e_f(1) = 2n$.
Thus, $|e_f(0) - e_f(1)| \leq 1$
Hence, Fl_n is sum divisor cordial graph. □

Example 2.14. *A sum divisor cordial labeling of Fl_4 is shown in Figure 2.7.*

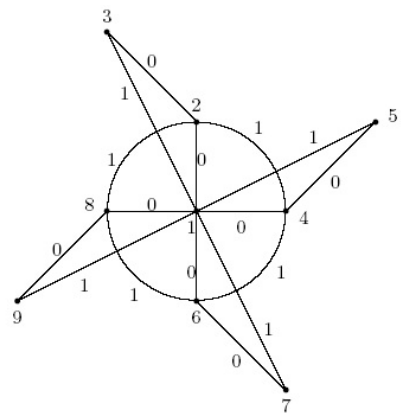

Figure 2.7

Theorem 2.15. *The jewel J_n is sum divisor cordial graph.*

Proof. Let $G = J_n$. Let $V(G) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and $E(G) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$. Then G is of order $n+4$ and size $2n+5$. Define $f : V(G) \to \{1, 2, \cdots, n+4\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(x) = 3;$$
$$f(y) = 4;$$
$$f(u_i) = i + 4; 1 \leq i \leq n.$$

Then, the induced edge labels are

$$f^*(ux) = 1;$$
$$f^*(uy) = 0;$$
$$f^*(xy) = 0;$$
$$f^*(vx) = 0;$$
$$f^*(vy) = 1;$$
$$f^*(uu_{2i-1}) = 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(uu_{2i}) = 0; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(vu_{2i-1}) = 0; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$

$f^*(vu_{2i}) = 1; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$
We observe that, $e_f(0) = n + 3$ and $e_f(1) = n + 2$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, J_n is sum divisor cordial graph. □

Example 2.16. *A sum divisor cordial labeling of J_4 is shown in Figure 2.8.*

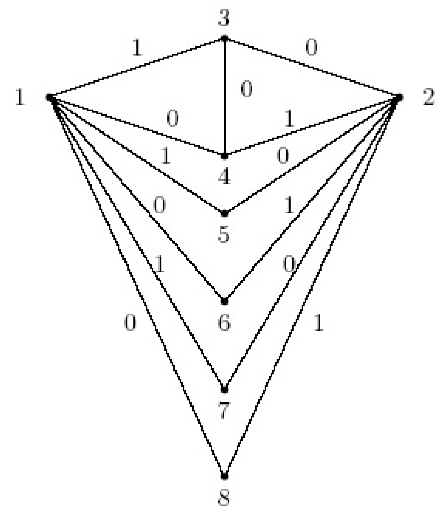

Figure 2.8

Theorem 2.17. *The crown $C_n \odot K_1$ is sum divisor cordial graph.*

Proof. Let $G = C_n \odot K_1$. Let $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{u_i u_{i+1} : 1 \leq i \leq n-1; u_n u_1, u_i v_i : 1 \leq i \leq n\}$. Then G is of order $2n$ and size $2n$. Define $f : V(G) \to \{1, 2, \cdots, 2n\}$ as follows:

$f(u_i) = 2i; 1 \leq i \leq n$
$f(v_i) = 2i - 1; 1 \leq i \leq n$

Then, the induced edge labels are

$f^*(u_i u_{i+1}) = 1; 1 \leq i \leq n - 1$
$f^*(u_n u_1) = 1;$
$f^*(u_i v_i) = 0; 1 \leq i \leq n$

We observe that, $e_f(0) = e_f(1) = n$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $C_n \odot K_1$ is sum divisor cordial graph. □

Example 2.18. *A sum divisor cordial labeling of $C_4 \odot K_1$ is shown in Figure 2.9.*

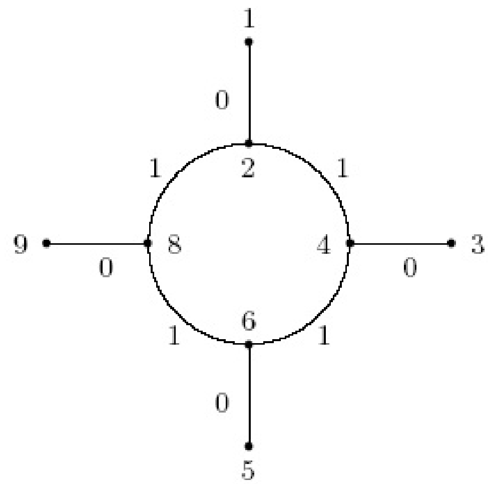

Figure 2.9

Theorem 2.19. *The gear G_n is sum divisor cordial graph.*

Proof. Let $G = G_n$. Let $V(G) = \{v, u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{vv_i, v_iu_i : 1 \leq i \leq n; u_iv_{i+1} : 1 \leq i \leq n-1; u_nv_1\}$. Then G is of order $2n+1$ and size $3n$. Define $f : V(G) \rightarrow \{1, 2, \cdots, 2n+1\}$ as follows:

$$f(v) = 1;$$
$$f(v_{2i-1}) = 4i - 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f(v_{2i}) = 4i; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f(u_{2i-1}) = 4i - 2; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f(u_{2i}) = 4i + 1; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$

Then, the induced edge labels are

$$f^*(vv_{2i-1}) = 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(vv_{2i}) = 0; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(v_iu_i) = 0; 1 \leq i \leq n$$
$$f^*(u_iv_{i+1}) = 1; 1 \leq i \leq n-1$$

$$f^*(u_nv_1) = \begin{cases} 0 & \text{if } n \text{ is odd} \\ 1 & \text{if } n \text{ is even} \end{cases}$$

We observe that, $e_f(0) = \left\lceil \frac{3n}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor$
Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, G_n is sum divisor cordial graph. □

Example 2.20. *A sum divisor cordial labeling of G_8 is shown in Figure 2.10.*

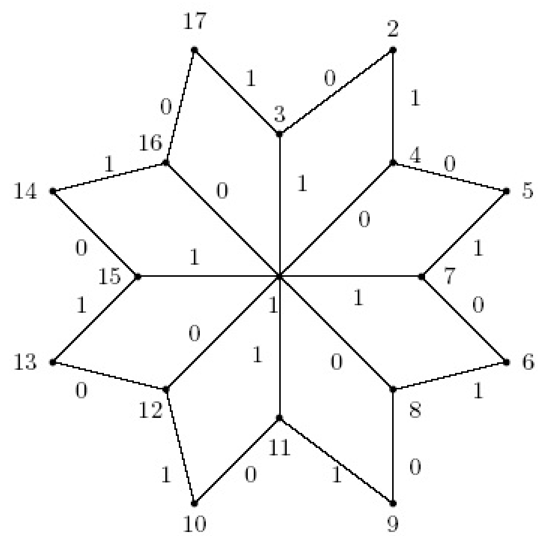

Figure 2.10

Theorem 2.21. *The graph $K_{1,3} * K_{1,n}$ is sum divisor cordial graph.*

Proof. Let $G = K_{1,3} * K_{1,n}$. Let $V(G) = \{x, u, v, w, u_i, v_i, w_i : 1 \leq i \leq n\}$ and $E(G) = \{xu, xv, xw, uu_i, vv_i, ww_i : 1 \leq i \leq n\}$. Then G is of order $3n + 4$ and size $3n + 3$. Define $f : V(G) \to \{1, 2, \cdots, 3n + 4\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(w) = 4;$$
$$f(x) = 3;$$
$$f(u_i) = 3i + 2; 1 \leq i \leq n$$
$$f(v_i) = 3i + 3; 1 \leq i \leq n$$
$$f(w_i) = 3i + 4; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(xu) = 1;$$

$$f^*(xv) = 0;$$
$$f^*(xw) = 0;$$
$$f^*(uu_{2i-1}) = 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(uu_{2i}) = 0; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(vv_{2i-1}) = 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(vv_{2i}) = 0; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(ww_{2i-1}) = 0; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(ww_{2i}) = 1; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$

We observe that,

$$e_f(0) = \begin{cases} \frac{3n+3}{2} & \text{if } n \text{ is odd} \\ \frac{3n+4}{2} & \text{if } n \text{ is even} \end{cases}$$

$$e_f(1) = \begin{cases} \frac{3n+3}{2} & \text{if } n \text{ is odd} \\ \frac{3n+2}{2} & \text{if } n \text{ is even} \end{cases}$$

Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, $K_{1,3} * K_{1,n}$ is sum divisor cordial graph. □

Example 2.22. *A sum divisor cordial labeling of $K_{1,3} * K_{1,5}$ is shown in Figure 2.11.*

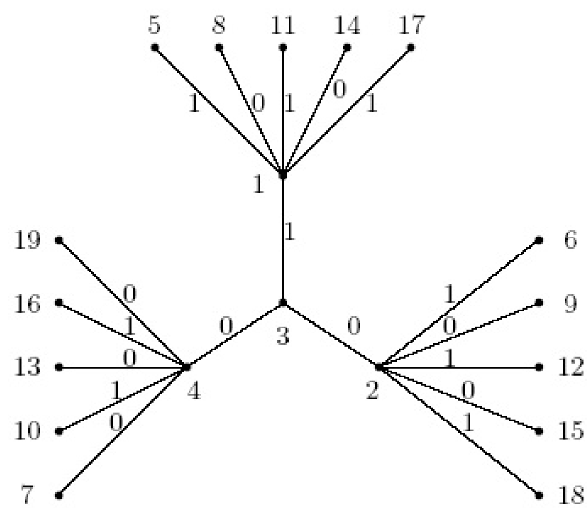

Figure 2.11

Theorem 2.23. *The graph $B^2_{n,n}$ is sum divisor cordial graph.*

Proof. Let $G = B^2_{n,n}$. Let $V(G) = \{u, v, u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{uv, vv_i, uu_i, u_iv, v_iu : 1 \leq i \leq n\}$. Then G is of order $2n+2$ and size $4n+1$. Define $f : V(G) \to \{1, 2, \cdots, 2n+2\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(u_{2i-1}) = 4i - 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f(u_{2i}) = 4i + 2; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f(v_{2i-1}) = 4i; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f(v_{2i}) = 4i + 1; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$

Then, the induced edge labels are

$$f^*(uv) = 0;$$
$$f^*(uu_{2i-1}) = 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(uu_{2i}) = 0; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(vv_{2i-1}) = 1; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(vv_{2i}) = 0; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(vu_{2i-1}) = 0; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(vu_{2i}) = 1; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(uv_{2i-1}) = 0; 1 \leq i \leq \lceil \tfrac{n}{2} \rceil$$
$$f^*(uv_{2i}) = 1; 1 \leq i \leq \lfloor \tfrac{n}{2} \rfloor$$

We observe that, $e_f(1) = 2n$ and $e_f(0) = 2n+1$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, the graph $B^2_{n,n}$ is sum divisor cordial graph. □

Example 2.24. *A sum divisor cordial labeling of $B^2_{4,4}$ is shown in Figure 2.12.*

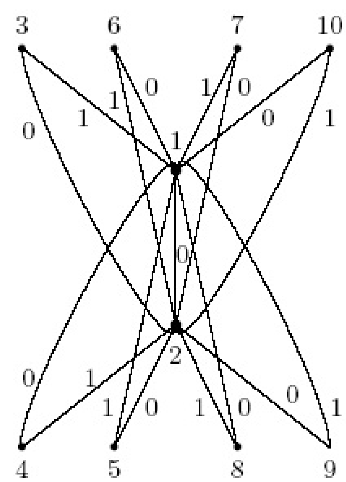

Figure 2.12

Theorem 2.25. *The graph $S(K_{1,n})$ is sum divisor cordial graph.*

Proof. Let $G = S(K_{1,n})$. Let $V(G) = \{v, v_i, u_i : 1 \leq i \leq n\}$ and $E(G) = \{vv_i, v_iu_i : 1 \leq i \leq n\}$. Then G is of order $2n+1$ and size $2n$. Define $f : V(G) \to \{1, 2, \cdots, 2n+1\}$ as follows:

$$f(v) = 1;$$
$$f(v_i) = 2i+1; 1 \leq i \leq n$$
$$f(u_i) = 2i; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(vv_i) = 1; 1 \leq i \leq n$$
$$f^*(v_iu_i) = 0; 1 \leq i \leq n$$

We observe that, $e_f(0) = e_f(1) = n$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.

Hence, $S(K_{1,n})$ is sum divisor cordial graph. □

Example 2.26. *A sum divisor cordial labeling of $S(K_{1,5})$ is shown in Figure 2.13.*

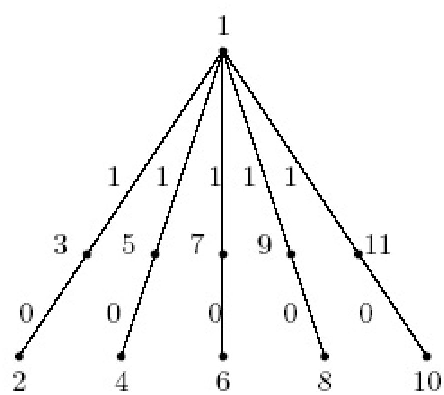

Figure 2.13

3. conclusion

All the graphs are not sum divisor cordial graphs. It is very interesting and challenging as well to investigate sum divisor cordial labeling for the graph or graph families which admit sum divisor cordial labeling. Here we have proved path, comb, star, complete bipartite, $K_2 + mK_1$, bistar, jewel, crown, flower, gear, subdivision of the star, $K_{1,3} * K_{1,n}$ and square graph of $B_{n,n}$ are sum divisor cordial graphs. In the subsequent paper, we will prove that total graph of the path, square graph of the path, shadow graph of the path and alternative triangular snake are sum divisor cordial graphs. Also, we will prove book, one point union of cycles, triangular ladder related graphs are sum divisor cordial graphs.

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